Zadatak 16. Riješi jednadžbu $f'(x)=0$, ako je:

1) $f(x)=\ln^2 x-\ln x-1$;

2) $f(x)=\ln(x^2+x+1)$;

3) $f(x)=x\cdot\ln x$;

4) $f(x)=x^2\cdot\ln x$;

5) $f(x)=\ln\sin x$;

6) $f(x)=\ln(\operatorname{tg} x+\operatorname{ctg} x)$.

Rješenje. **1)** $f'(x)=(\ln^2 x-\ln x-1)'=2\ln x\cdot\frac{1}{x}-\frac{1}{x}=\frac{1}{x}(2\ln x-1)$.

$\frac{1}{x}(2\ln x-1)=0 \implies 2\ln x-1=0 \implies \ln x=\frac{1}{2} \implies \ln x=\ln e^{\frac{1}{2}} \implies x=\sqrt{e}$;

2) $f'(x)=[\ln(x^2+x+1)]=\frac{1}{x^2+x+1}\cdot(2x+1)=\frac{2x+1}{x^2+x+1}$.

$\frac{2x+1}{x^2+x+1}=0 \implies 2x+1=0 \implies 2x=-1 \implies x=-\frac{1}{2}$;

3) $f'(x)=(x\cdot\ln x)'=\ln x+1$.

$\ln x+1=0 \implies \ln x=-1 \implies \ln x=\ln e^{-1} \implies x=\frac{1}{e}$;

4) $f'(x)=(x^2\cdot\ln x)'=2x\ln x+x=x(2\ln x+1)$.

$x(2\ln x+1)=0 \implies 2\ln x+1=0 \implies \ln x=-\frac{1}{2} \implies \ln x=\ln e^{-\frac{1}{2}} \implies x=\frac{1}{\sqrt{e}}$;

5) $f'(x)=(\ln\sin x)'=\frac{1}{\sin x}\cdot\cos x=\frac{\cos x}{\sin x}$.

$\frac{\cos x}{\sin x}=0 \implies \cos x=0 \implies x=\frac{\pi}{2}+k\cdot\pi$;

6) $f'(x)=[\ln(\operatorname{tg} x+\operatorname{ctg} x)]'=\frac{1}{\operatorname{tg} x+\operatorname{ctg} x}\cdot\left(\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right)=\frac{1}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}\cdot$

$\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}=\frac{\sin x\cos x}{\sin^2 x+\cos^2 x}\cdot\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cdot\cos^2 x}=\frac{\sin^2 x-\cos^2 x}{\sin x\cos x}=\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}$.

$\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}=0 \implies \frac{\sin x}{\cos x}=\frac{\cos x}{\sin x}=0 \implies \sin x=\cos x \implies x=\frac{\pi}{4}+k\cdot\pi, k\in\mathbf{Z}$.