Zadatak 3. Da li funkcija $f(x) = \operatorname{tg} x \cdot \operatorname{tg}\left(\frac{\pi}{4} - x\right)$ u $x = \frac{\pi}{6}$ raste ili pada?

Rješenje.

$$f'(x) = \frac{1}{\cos^2 x}\operatorname{tg}\left(\frac{\pi}{4} - x\right) - \operatorname{tg} x \frac{1}{\cos^2\left(\frac{\pi}{4} - x\right)} = \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos^2 x \cos\left(\frac{\pi}{4} - x\right)} -$$

$$\frac{\sin x}{\cos x \cos^2\left(\frac{\pi}{4} - x\right)} = \frac{\sin\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - x\right) - \sin x \cos x}{\cos^2 x \cos^2\left(\frac{\pi}{4} - x\right)}$$

$$= \frac{\frac{1}{2}\sin\left(\frac{\pi}{2} - 2x\right) - \frac{1}{2}\sin 2x}{\left[\cos x \cos\left(\frac{\pi}{4} - x\right)\right]^2} = \frac{1}{2} \cdot \frac{\sin\left(\frac{\pi}{2} - 2x\right) - \sin 2x}{\left[\cos x \cos\left(\frac{\pi}{4} - x\right)\right]^2};$$

$$f'\left(\frac{\pi}{6}\right) = \frac{1}{2} \cdot \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - \sin\frac{\pi}{3}}{\left[\cos\frac{\pi}{6}\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\right]^2} = \frac{1}{2} \cdot \frac{\sin\frac{\pi}{6} - \sin\frac{\pi}{3}}{\left[\cos\frac{\pi}{6}\cos\frac{\pi}{12}\right]^2}$$

$$= \frac{1}{2} \cdot \frac{\frac{1}{2} - \frac{\sqrt{3}}{2}}{\left[\cos\frac{\pi}{6}\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\right]^2} = \frac{1}{4} \cdot \underbrace{\frac{1 - \sqrt{3}}{\left[\cos\frac{\pi}{6}\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\right]^2}}_{>0} < 0.$$ Funkcija pada.