Zadatak 9. Ispitaj monotonost funkcija:

1) $f(x)=\dfrac{x^3-2x^2}{e^x}$; **2)** $f(x)=\dfrac{\ln^2 x+2\ln x}{x}$;

3) $f(x)=\dfrac{x^3-x^2}{e^{-x}}$; **4)** $f(x)=\dfrac{2\ln^2 x+3\ln x}{x}$.

Rješenje. **1)** $f'(x)=\dfrac{(3x^2-4x)e^x-(x^3-2x^2)e^x}{e^{2x}}=\dfrac{3x^2-4x-x^3+2x^2}{e^x}$ $=-\dfrac{x^3-5x^2+4x}{e^x}=-\dfrac{x(x-1)(x-4)}{e^x}$. Budući da je $e^x>0$ za $\forall x\in\mathbf{R}$, promatramo samo brojnik $x(x-1)(x-4)$. Stacionarne točke su 0, 1 i 4. $x(x-1)(x-4)<0 \implies x\langle-\infty,0\rangle\cup\langle1,4\rangle$; $x(x-1)(x-4)>0 \implies \langle0,1\rangle\cup\langle4+\infty\rangle$. Za $\langle0,1\rangle\cup\langle4,+\infty\rangle$ funkcija pada, a na $\mathbf{R}^-\cup\langle1,4\rangle$ raste.

2) $f'(x)=\dfrac{\left(2\ln x\cdot\dfrac{1}{x}+\dfrac{2}{x}\right)\cdot x-(\ln^2 x+2\ln x)\cdot 1}{x^2}=\dfrac{2+2\ln x-\ln^2 x-2\ln x}{x^2}=$ $\dfrac{(\sqrt{2}-\ln x)(\sqrt{2}+\ln x)}{x^2}$. stacionarne točke su $e^{-\sqrt{2}}$ i $e^{\sqrt{2}}$. Promatramo $(\sqrt{2}-\ln x)(\sqrt{2}+\ln x)<0 \implies x\in\langle0,e^{-\sqrt{2}}\rangle\cup\langle e^{\sqrt{2}},+\infty\rangle$; $(\sqrt{2}-\ln x)(\sqrt{2}+\ln x)>0 \implies x\in\langle e^{-\sqrt{2}},e^{\sqrt{2}}\rangle$. Funkcija raste na intervalu $\langle e^{-\sqrt{2}},e^{\sqrt{2}}\rangle$, a pada na $\mathbf{R}^+\setminus\langle e^{-\sqrt{2}},e^{\sqrt{2}}\rangle$.

3) $f'(x)=\dfrac{(3x^2-2x)e^{-x}+(x^3-x^2)e^{-x}}{e^{-2x}}=\dfrac{3x^2-2x+x^3-x^2}{e^{-x}}=\dfrac{x(x^2+2x-2)}{e^{-x}}$ $=\dfrac{x[(x+1)^2-3]}{e^{-x}}=e^x x(x+1-\sqrt{3})(x+1+\sqrt{3})$. Stacionarne točke su 0, $\sqrt{3}-1$ i $-1-\sqrt{3}$. $x(x+1-\sqrt{})(x+1+\sqrt{3})<0 \implies x\in\langle-\infty,-\sqrt{3}-1\rangle\cup\langle0,\sqrt{3}-1\rangle$; $x(x+1-\sqrt{3})(x+1+\sqrt{3})>0 \implies x\in\langle-\sqrt{3}-1,0\rangle\cup\langle\sqrt{3}-1,+\infty\rangle$. Funkcija pada na $\langle-\infty,-1-\sqrt{3}\rangle\cup\langle0,-1+\sqrt{3}\rangle$, a raste na $\langle-1-\sqrt{3},0\rangle\cup\langle-1+\sqrt{3},+\infty\rangle$.

4) $f'(x)=\dfrac{\left(4\ln x\cdot\dfrac{1}{x}+\dfrac{3}{x}\right)\cdot x-2\ln^2 x-3\ln x}{x^2}=\dfrac{-2\ln^2 x+\ln x+3}{x^2}=$ $\dfrac{-2\ln^2 x-2\ln x+3\ln x+3}{x^2}=\dfrac{(3-2\ln x)(\ln x+1)}{x^2}$. Stacionarne točke su $x_1=e^{-1}$ i $x_2=e^{\frac{3}{2}}$. $(3-2\ln x)(\ln x+1)<0 \implies x\in\left\langle0,\dfrac{1}{e}\right\rangle\cup\langle e\sqrt{e},+\infty\rangle$; $(3-2\ln x)(\ln x+1)>0 \implies x\in\left\langle\dfrac{1}{e},e\sqrt{e}\right\rangle$. Funkcija raste na $\left\langle\dfrac{1}{e},e\sqrt{e}\right\rangle$, a pada na $\left\langle0,\dfrac{1}{e}\right\rangle\cup\langle e\sqrt{e},+\infty\rangle$.