Zadatak 10. Odredi intervale monotonosti funkcija:

1) $f(x)=\dfrac{\ln x-x}{x}$;

2) $f(x)=\dfrac{3}{2}\log^2 x+\log^3 x$;

3) $f(x)=x\cdot e^{-2x}$;

4) $f(x)=\ln^4 x-2\ln^2 x$.

Rješenje. **1)** $f'(x)=\dfrac{\left(\frac{1}{x}-1\right)\cdot x-(\ln x-x)\cdot 1}{x^2}=\dfrac{1-x-\ln x+x}{x^2}=\dfrac{1-\ln x}{x^2}$. $1-\ln x<0 \implies x\in\langle e,+\infty\rangle$; $1-\ln x>0 \implies x\in\langle 0,e\rangle$. Funkcija raste na $\langle 0,e\rangle$, a pada na $\langle e,+\infty\rangle$.

2) $f'(x)=3\log x\cdot\dfrac{1}{x\log 10}+3\log^2 x\cdot\dfrac{1}{x\ln 10}=\dfrac{3}{\ln 10}\cdot\dfrac{\log x}{x}(1+\log x)$. Stacionarne točke su 1 i $\dfrac{1}{10}$. $\log x(1+\log x)<0 \implies x\in\left\langle\dfrac{1}{10},1\right\rangle$; $\log x(1+\log x)>0 \implies x\in\left\langle 0,\dfrac{1}{10}\right\rangle\cup\langle 1,+\infty\rangle$. Funkcija raste na $\left\langle 0,\dfrac{1}{10}\right\rangle\cup\langle 1,+\infty\rangle$, a pada na $\left\langle\dfrac{1}{10},1\right\rangle$.

3) $f'(x)=e^{-2x}+x^e-2x(-2)=(1-2x)e^{-2x}$. e^{-2x} je uvijek veće od 0 pa promatramo samo izraz u zagradi. $1-2x<0 \implies x\in\left\langle\dfrac{1}{2},+\infty\right\rangle$; $1-2x>0 \implies x\in\left\langle-\infty,\dfrac{1}{2}\right\rangle$. Funkcija raste na $\left\langle-\infty,\dfrac{1}{2}\right\rangle$, a pada na $\left\langle\dfrac{1}{2},+\infty\right\rangle$.

4) $f'(x)=\dfrac{4\ln^3 x}{x}-\dfrac{4\ln x}{x}=\dfrac{4\ln x}{x}(\ln x-1)(\ln x+1)$. Stacionarne točke su 1, e i $\dfrac{1}{e}$. Promatramo samo izraz $\ln x(\ln x-1)(\ln x+1)$ jer je $x>0$ za $\forall x\in\mathbf{R}$ zbog definicije funkcije prirodnog logaritma. $\ln x(\ln x-1)(\ln x+1)<0 \implies x\in\left\langle 0,\dfrac{1}{e}\right\rangle\cup\langle 1,e\rangle$; $\ln x(\ln x-1)(\ln x+1)>0 \implies x\in\left\langle\dfrac{1}{e},1\right\rangle\cup\langle e,+\infty\rangle$. Funkcija pada na $\left\langle 0,\dfrac{1}{e}\right\rangle\cup\langle 1,e\rangle$, a raste na $\left\langle\dfrac{1}{e},1\right\rangle\cup\langle e,+\infty\rangle$.