

Zadatak 24. Odredi realne brojeve a i b tako da funkcija $f(x) = \frac{\sin^2 x}{a - b \cos x}$ ima ekstremnu vrijednost $\frac{1}{4}$ za $x = \frac{\pi}{3}$.

Rješenje. $f'(x) = \frac{\sin 2x(a - b \cos x) - \sin^2 x \cdot b \sin x}{(a - b \cos x)^2}$, $\sin 2x(a - b \cos x) - \sin 2x \cdot b \sin x = 0$.

$$\frac{\sqrt{3}}{2} \left(a - b \cdot \frac{1}{2} \right) - b \cdot \frac{3\sqrt{3}}{8} = 0$$

$$\frac{\sqrt{3}}{4} (2a - b) - \frac{3\sqrt{3}}{8} b = 0 \cdot 8$$

$$2\sqrt{3}(2a - b) - 3\sqrt{3}b = 0$$

$$4a\sqrt{3} - 2b\sqrt{3} - 3b\sqrt{3} = 0$$

$$4a\sqrt{3} - 5b\sqrt{3} = 0 \quad / : \sqrt{3}$$

$$4a - 5b = 0$$

$$f\left(\frac{\pi}{3}\right) = \frac{\frac{3}{4}}{a - b \cdot \frac{1}{2}} = \frac{1}{4},$$

$$\frac{\frac{3}{4}}{2a - b} = \frac{1}{4}$$

$$\frac{3}{4a - 2b} = \frac{1}{4}$$

$$4a - 2b = 12$$

$$2a - 6 = b.$$

$$4a - 5(2a - 6) = 0 \implies -6a = -30 \implies a = 5, b = 4.$$