Zadatak 30. Dokaži da je najmanja vrijednost funkcije $f(x) = \cos x \cdot \sin 2x$ na intervalu $\langle -\pi, \pi \rangle$ veća od $-\frac{7}{9}$.

Rješenje. $f(x) = 2\sin x \cos^2 x$, $f'(x) = 2\cos^3 x - 4\sin^2 x \cos x = 2\cos x(\cos^2 x - 2\sin^2 x) = 2\cos x(3\cos^2 x - 2)$.

$f'(x) = 0 \implies x_1 = \frac{\pi}{2}$, $x_2 = -\frac{\pi}{2}$;

$\cos^2 x = \frac{2}{3} \implies \sin^2 x = \frac{1}{3} \implies \operatorname{tg}^2 x = \frac{1}{2} \implies \operatorname{tg} x = \pm\frac{\sqrt{2}}{2} \implies x_3 = 35^\circ 15' 51.8''$, $x_4 = -35^\circ 15' 51.8''$, $x_5 = -144^\circ 44' 8.2''$, $x_6 = 144^\circ 44' 8.2''$.

$f(x_1) = 0$, $f(x_2) = 0$, $f(x_3) > 0$, $f(x_4) > 0$, $f(x_5) = 0.9428 > -\frac{7}{9} = -0.7\dot{7}$, $f(x_0) > 0$.