Zadatak 5. Grafički prikaži sljedeće funkcije:

1) $f(x)=x^2\sqrt{2-x}$; **2)** $f(x)=\sqrt[3]{(x-1)^2}$;

3) $f(x)=x+e^{-x}$; **4)** $f(x)=e^{\frac{1}{x}}$.

Rješenje. **1)** $f(x)=x^2\sqrt{2-x}=x^2(2-x)^{\frac{1}{2}}$,

$$2-x\geqslant 0 \implies -x\geqslant -2 \implies x\leqslant 2 \implies D_f=\langle -\infty, 2]$$

$$\lim_{x\to-\infty} f(x)=+\infty, \quad \lim_{x\to-\infty}\frac{f(x)}{x}=\infty \implies \text{nema asimptota}$$

$$f(x)=0 \implies x_1=0, \quad x_2=2 \quad \text{nultočka}$$

x	$\langle -\infty, 0\rangle$	$\langle 0, 2\rangle$
$f(x)$	$+$	$+$

$$f'(x)=2x(2-x)^{\frac{1}{2}}+x^2\cdot\frac{1}{2}(2-x)^{-\frac{1}{2}}(-1)=2x\sqrt{2-x}-\frac{x^2}{2\sqrt{2-x}}$$

$$f'(x)=\frac{4x(2-x)-x^2}{2\sqrt{2-x}}=\frac{8x-5x^2}{2\sqrt{2-x}}=\frac{x(8-5x)}{2\sqrt{2-x}}$$

$$f'(x)=0 \implies x_1=0, \quad x_2=\frac{8}{5}, \quad f\left(\frac{8}{5}\right)\approx 1.62$$

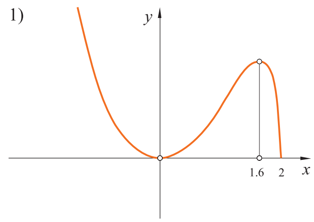
x	$\langle -\infty, 0\rangle$	$\left\langle 0, \frac{8}{5}\right\rangle$	$\left\langle \frac{8}{5}, 2\right\rangle$
$f'(x)$	$-$	$+$	$-$

$$f'(x)=\left(4x-\frac{5}{2}x^2\right)(2-x)^{-\frac{1}{2}}$$

$$f''(x)=(4-5x)(2-x)^{-\frac{1}{2}}+\left(4x-\frac{5}{2}x^2\right)\left(-\frac{1}{2}\right)(2-x)^{-\frac{3}{2}}(-1)$$

$$=\frac{4-5x}{\sqrt{2-x}}+\frac{8x-5x^2}{4\sqrt{(2-x)^3}}=\frac{(4-5x)\cdot 4(2-x)+8x-5x^2}{4\sqrt{(2-x)^3}}=\frac{15x^2-48x+32}{4\sqrt{(2-x)^3}}$$

$$f''(0)>0 \implies m(0,0), \quad f''\left(\frac{8}{5}\right)<0 \implies M\left(\frac{8}{5}, 1.62\right)$$



2) $f(x) = \sqrt[3]{(x-1)^2} = (x-1)^{\frac{2}{3}}$,

$D_f = \mathbf{R}, \quad f(x) = 0 \implies x = 1$ nultočka

$\lim\limits_{x\to\pm\infty} f(x) = +\infty, \quad \lim\limits_{x\to\infty} \frac{f(x)}{x} = 0 \implies$ nema asimptota

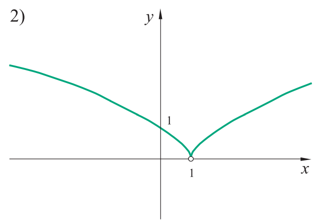
x	$\langle -\infty, 1\rangle$	$\langle 1, +\infty\rangle$
$f(x)$	$+$	$+$

$f(0) = 1$

$$f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x-1}}$$

x	$\langle -\infty, 1\rangle$	$\langle 1, +\infty\rangle$
$f'(x)$	$-$	$+$

$$f''(x) = -\frac{2}{9}(x-1)^{-\frac{4}{3}} < 0, \quad \forall x \in D_f$$



3) $f(x) = x + e^{-x} > 0$, $\forall x \in \mathbf{R}$,

$D_f = \mathbf{R}$

$\lim\limits_{x\to\pm\infty} (x + e^{-x}) = \pm\infty$

$\lim\limits_{x\to-\infty} \frac{x+e^{-x}}{x} = +\infty, \quad \lim\limits_{x\to+\infty} \frac{x+e^{-x}}{x} = 1$

$\lim\limits_{x\to+\infty} (x + e^{-x} - x) = \lim\limits_{x\to+\infty} e^{-x} = 0$

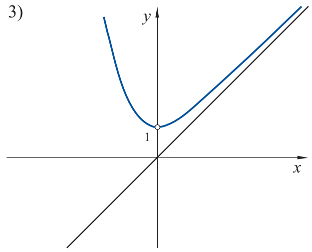
$\implies y = x$ desna kosa asimptota

$f'(x) = 1 - e^{-x}, \quad f'(x) = 0 \implies x = 0, \quad f(0) = 1$

$f''(x) = e^{-x} > 0, \quad \forall x \in D_f, \quad f''(0) > 0 \implies m(0,1)$

x	$\langle -\infty, 0\rangle$	$\langle 0, +\infty\rangle$
$f'(x)$	$-$	$+$

3)



4) $f(x) = e^{\frac{1}{x}}$,

$D_f = \mathbf{R}^*, \quad f(x) > 0, \quad \forall x \in \mathbf{R}$

$\lim\limits_{x \to 0^-} e^{\frac{1}{x}} = 0, \quad \lim\limits_{x \to 0^+} e^{\frac{1}{x}} = +\infty, \quad x = 0$ vertikalna asimptota

$\lim\limits_{x \to \pm\infty} e^{\frac{1}{x}} = 1 \implies y = 1$ horizontalna asimptota

$f'(x) = -\frac{1}{x^2} e^{\frac{1}{x}} < 0, \quad \forall x \in D_f$

$$f''(x) = \frac{2}{x^3} e^{\frac{1}{x}} - \frac{1}{x^2}\left(-\frac{1}{x^2}\right) e^{\frac{1}{x}} = \left(\frac{2}{x^3} + \frac{1}{x^4}\right) e^{\frac{1}{x}} = \frac{2x+1}{x^4} e^{\frac{1}{x}}$$

$f''(x) = 0 \implies x = -\frac{1}{2}$

$f\left(-\frac{1}{2}\right) = e^{-2} = \frac{1}{e^2}$

$\implies \left(-\frac{1}{2}, \frac{1}{e^2}\right)$ infleksija

x	$\left\langle -\infty, -\frac{1}{2} \right\rangle$	$\left\langle -\frac{1}{2}, 0 \right\rangle$	$\langle 0, +\infty \rangle$
$f''(x)$	$-$	$+$	$+$

4)