

Zadatak 14. Koji stožac s opsegom osnovog presjeka 4 dm ima najveći volumen?

Rješenje.

$$\begin{aligned}2s + 2r = 4 &\implies s + r = 2 \implies s = 2 - r, r^2 + h^2 = s^2 \implies r^2 + h^2 = \\4 - 4r + r^2 &\implies h^2 = 4 - 4r \implies r = 1 - \frac{h^2}{4}. \text{ Volumen stošca je} \\V^2(r) &= \left[\frac{1}{3} r^2 \pi \cdot h \right]^2 = \frac{1}{9} r^4 \pi^2 h^2 = \frac{1}{9} r^4 \pi^2 (s^2 - r^2) = \frac{1}{9} r^4 \pi^2 [(2-r)^2 - r^2] = \\&= \frac{1}{9} r^4 \pi^2 (4 - 4r + r^2 - r^2) = \frac{4}{9} r^4 \pi^2 - \frac{4}{9} r^5 \pi^2. \text{ Deriviramo li tu funkciju dobit} \\&\text{ćemo } V^{2'}(r) = \frac{16}{9} r^3 \pi^2 - \frac{20}{9} r^4 \pi^2 = \frac{4}{9} r^3 \pi^2 (4 - 5r). \text{ Izjednačimo to s nulom} \\&\frac{4}{9} r^3 \pi^2 (4 - 5r) = 0 \implies r = \frac{4}{5}, s = \frac{6}{5}, h = \frac{2\sqrt{5}}{5}.\end{aligned}$$