

Zadatak 6. Odredi derivaciju funkcije $f \circ f$ ako je:

- | | |
|---------------------------|--------------------------------|
| 1) $f(x) = -2x + 3$; | 2) $f(x) = x^2 - x$; |
| 3) $f(x) = \frac{1}{x}$; | 4) $f(x) = \frac{2x-1}{x+2}$; |
| 5) $f(x) = \sin x$; | 6) $f(x) = \ln \frac{x}{2}$; |
| 7) $f(x) = x^n$; | 8) $f(x) = \sqrt[3]{x}$. |

Rješenje.

$$1) f(f(x))' = [-2(-2x+3)+3]' = (4x-6+3)' = (4x-3)' = 4;$$

$$2) f(f(x))' = [(x^2-x)^2 - (x^2-x)]' = (x^4-2x^3+x^2-x^2+x)' = (x^4-2x^3+x)' = 4x^3-6x^2+1;$$

$$3) f(f(x))' = \left(\frac{1}{\frac{1}{x}}\right)' = (x)' = 1;$$

$$4) f(f(x))' = \left(\frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}\right)' = \left(\frac{\frac{4x-2-x-2}{x+2}}{\frac{2x-1+2x+4}{x+2}}\right)' = \left(\frac{3x-4}{4x+3}\right)' = \frac{(3x-4)'(4x+3) - (3x-4)(4x+3)'}{(4x+3)^2} = \frac{12x+9-12x+16}{(4x+3)^2} = \frac{25}{(4x+3)^2};$$

$$5) f(f(x))' = (\sin(\sin x))' = \cos(\sin x) \cdot \cos x;$$

$$6) f(f(x))' = \left[\ln\left(\frac{\ln \frac{x}{2}}{2}\right)\right]' = \frac{1}{\ln\left(\frac{x}{2}\right)} \cdot \left(\frac{\ln\left(\frac{x}{2}\right)}{2}\right)' = \frac{2}{\ln\left(\frac{x}{2}\right)} \cdot \frac{2 \cdot \frac{1}{x} \cdot \frac{1}{2}}{4} =$$

$$\frac{1}{\ln\left(\frac{x}{2}\right)} \cdot \frac{\frac{2}{x}}{2} = \frac{1}{x \ln\left(\frac{x}{2}\right)};$$

$$7) f(f(x))' = [(x^n)^n]' = (x^{n^2})' = n^2 \cdot x^{n^2-1};$$

$$8) f(f(x))' = (\sqrt[3]{\sqrt[3]{x}})' = (x^{\frac{1}{9}})' = \frac{1}{9}x^{-\frac{8}{9}} = \frac{1}{9} \cdot \frac{1}{x^{\frac{8}{9}}} = \frac{1}{9\sqrt[9]{x^8}}.$$