

Zadatak 7. Dane su funkcije f i g . Odredi derivaciju funkcije $f \circ g$ ako je:

$$1) f(x) = \ln \frac{x}{2}, g(x) = e^{1-2x};$$

$$2) f(x) = \ln \sqrt{1-x^2}, g(x) = \frac{1}{e^x};$$

$$3) f(x) = \ln \frac{1-x}{1+x}, g(x) = \frac{1}{e^x};$$

$$4) f(x) = \ln \frac{x}{1-x}, g(x) = e^{2x}.$$

Rješenje.

$$1) f(g(x))' = \left[\ln \left(\frac{e^{1-2x}}{2} \right) \right]' = \frac{1}{e^{1-2x}} \cdot \left(\frac{e^{1-2x}}{2} \right)' = \frac{2}{e^{1-2x}} \cdot \frac{2(e^{1-2x})'}{4} = \frac{e^{1-2x} \cdot (-2)}{e^{1-2x}} = -2;$$

$$2) f(g(x))' = \left[\ln \sqrt{1 - \left(\frac{1}{e^x} \right)^2} \right]' = \left[\ln \sqrt{1 - e^{-2x}} \right]' = \frac{1}{\sqrt{1 - e^{-2x}}} \cdot \frac{1}{2\sqrt{1 - e^{-2x}}} \cdot [-(-2e^{-2x})] = \frac{e^{-2x}}{1 - e^{-2x}} = \frac{\frac{1}{e^{2x}}}{1 - \frac{1}{e^{2x}}} = \frac{\frac{1}{e^{2x}}}{\frac{e^{2x} - 1}{e^{2x}}} = \frac{1}{e^{2x} - 1};$$

$$3) f(g(x))' = \left(\ln \frac{1 - \frac{1}{e^x}}{1 + \frac{1}{e^x}} \right)' = \left(\ln \frac{e^x - 1}{e^x + 1} \right)' = \left(\ln \frac{e^x - 1}{e^x + 1} \right)' = [\ln(e^x - 1) - \ln(e^x + 1)]' = \frac{1}{e^x - 1} \cdot e^x - \frac{1}{e^x + 1} \cdot e^x = e^x \cdot \left(\frac{1}{e^x - 1} - \frac{1}{e^x + 1} \right) = e^x \cdot \frac{e^x + 1 - e^x + 1}{e^{2x} - 1} = \frac{2e^x}{e^{2x} - 1};$$

$$4) f(g(x))' = \left(\ln \frac{e^{2x}}{1 - e^{2x}} \right)' = \frac{1}{e^{2x}} \cdot \left(\frac{e^{2x}}{1 - e^{2x}} \right)' = \frac{1 - e^{2x}}{e^{2x}} \cdot \frac{2e^{2x}(1 - e^{2x}) - e^{2x} \cdot (-2e^{2x})}{(1 - e^{2x})^2} = \frac{e^{2x}(2 - 2e^{2x} + 2e^{2x})}{e^{2x}(1 - e^{2x})} = \frac{2}{1 - e^{2x}}.$$