Zadatak 8. Odredi derivaciju reda n za funkcije:

1) $f(x)=\frac{1}{x+1}$; **2)** $f(x)=\ln x$;

3) $f(x)=\frac{x+1}{x-1}$; **4)** $f(x)=\sin x$;

5) $f(x)=\cos 3x$; **6)** $f(x)=e^{-2x}$.

Rješenje. **1)**

$$f'(x)=-1\cdot(x+1)^{-2}=\frac{-1}{(x+1)^2}$$
$$f''(x)=-1\cdot(-2)\cdot(x+1)^{-3}=\frac{2}{(x+1)^3}$$
$$f'''(x)=-1\cdot(-2)\cdot(-3)\cdot(x+1)^{-4}=\frac{-6}{(x+1)^4}$$
$$\vdots$$
$$f^n(x)=(-1)^n\cdot n!\cdot(x+1)^{-n-1}=(-1)^n\cdot\frac{n!}{(x+1)^{n+1}};$$

2)

$$f'(x)=\frac{1}{x}=x^{-1}$$
$$f''(x)=-1\cdot x^{-2}=\frac{-1}{x^2}$$
$$f'''(x)=(-1)\cdot(-2)\cdot x^{-3}=\frac{2}{x^3}$$
$$\vdots$$
$$f^n(x)=(-1)^{n-1}\cdot(n-1)!\cdot x^{-n}=(-1)^{n-1}\cdot\frac{(n-1)!}{x^n};$$

3)

$$f(x)=\frac{x+1-1+1}{x-1}=\frac{x-1+2}{x-1}=1+\frac{2}{x-1}=1+2(x-1)^{-1}$$
$$f'(x)=2\cdot(-1)(x-1)^{-2}=-\frac{2}{(x-1)^2}$$
$$f''(x)=2\cdot(-1)\cdot(-2)(x-1)^{-3}=\frac{4}{(x-1)^3}$$
$$f'''(x)=2\cdot(-1)\cdot(-2)\cdot(-3)(x-1)^{-4}=-\frac{12}{(x-1)^4}$$
$$\vdots$$
$$f^n(x)=2\cdot(-1)^n\cdot n!\cdot(x-1)^{-n-1}=2\cdot(-1)^n\cdot\frac{n!}{(x-1)^{n+1}}$$

4)

$$f'(x) = \cos x$$
$$f''(x) = -\sin x$$
$$f'''(x) = -\cos x$$
$$\vdots$$
$$f^n(x) = \begin{cases} \sin x, & n = 4k \\ \cos x, & n = 4k-3 \\ -\sin x, & n = 4k-2 \\ -\cos x, & n = 4k-1 \end{cases} \quad ; k \in \mathbf{N}$$

5)

$$f'(x) = -3\sin 3x$$
$$f''(x) = -3 \cdot 3\cos 3x = -9\cos 3x$$
$$f'''(x) = -3 \cdot 3 \cdot (-3)\sin 3x = 27\sin 3x$$
$$\vdots$$
$$f^n(x) = \begin{cases} 3^{4k} \cdot \cos 3x, & n = 4k \\ -(3^{4k-3})\sin 3x, & n = 4k-3 \\ -(3)^{4k-2}\cos 3x, & n = 4k-2 \\ 3^{4k-1}\sin 3x, & n = 4k-1 \end{cases} \quad ; k \in \mathbf{N}$$

6)

$$f'(x) = -2 \cdot e^{-2x}$$
$$f''(x) = -2 \cdot (-2) \cdot e^{-2x} = 4e^{-2x}$$
$$f'''(x) = -2 \cdot (-2) \cdot (-2) \cdot e^{-2x} = -8e^{-2x}$$
$$\vdots$$
$$f^n(x) = (-2)^n \cdot e^{-2x}.$$