

Zadatak 9. Riješi jednadžbu $f'(x) = 0$ ako je:

- 1) $f(x) = \frac{1}{2} \sin^2 x - \sin x + 4$;
- 2) $f(x) = \sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x$;
- 3) $f(x) = \cos x - \sqrt{3} \sin x - \cos 2x$;
- 4) $f(x) = \sqrt{2}(\sin x + \cos x) + x - \sin^2 x$.

Rješenje. 1) $f'(x) = \sin x \cdot \cos x - \cos x = 0 \implies \cos x(\sin x - 1) = 0 \implies \cos x = 0$ ili $\sin x = 1$. $x = \frac{\pi}{2} + k\pi, k \in \mathbf{Z}$.

2) $f'(x) = \cos x + \cos 2x + \cos 3x = \cos x + 2 \cos^2 x - 1 + 4 \cos^3 x - 3 \cos x = -2 \cos x + 2 \cos^2 x - 1 + 4 \cos^3 x = 2 \cos^2 x(2 \cos x + 1) - (2 \cos x + 1) = (2 \cos^2 x - 1)(2 \cos x + 1) = (\sqrt{2} \cos x - 1)(\sqrt{2} \cos x + 1)(2 \cos x + 1) = 0 \implies \cos x = \frac{\sqrt{2}}{2}$ ili $\cos x = -\frac{\sqrt{2}}{2}$ ili $\cos x = -\frac{1}{2}$. $x = \frac{\pi}{4} + 2k\pi, x = \frac{7\pi}{4} + 2k\pi$, $x = \frac{3\pi}{4} + 2k\pi, x = \frac{5\pi}{4} + 2k\pi, x = \frac{2\pi}{3} + 2k\pi$ i $x = \frac{4\pi}{3} + 2k\pi, k \in \mathbf{Z}$.

3) $f'(x) = -\sin x - \sqrt{3} \cos x + 2 \sin 2x = -2\left(\frac{1}{2} \cdot \sin x + \frac{\sqrt{3}}{2} \cdot \cos x + \sin 2x\right) = -2\left(\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} + \sin 2x\right) = -2\left[\sin\left(x + \frac{\pi}{3}\right) - \sin 2x\right] = -2 \cdot 2 \sin \frac{x + \frac{\pi}{3} + 2x}{2} \sin \frac{x + \frac{\pi}{3} - 2x}{2} = -4 \sin \frac{9x + \pi}{6} \sin \frac{\pi - 3x}{6} = 0 \implies \sin \frac{9x + \pi}{6} = 0$ ili $\sin \frac{\pi - 3x}{6} = 0$. $x = \frac{2\pi(1 + 3k)}{9}$ ili $x = \frac{-2\pi(1 + 3k)}{3}, k \in \mathbf{Z}$.

4) $f'(x) = \sqrt{2}(\cos x - \sin x) + 1 - 2 \sin x \cos x = \sqrt{2}(\cos x - \sin x) + (1 - \sin 2x) = \sqrt{2}(\cos x - \sin x) + (\cos x - \sin x)^2 = (\cos x - \sin x)(\sqrt{2} + \cos x - \sin x) = 0 \implies \cos x = \sin x$ ili $\sqrt{2} + \sqrt{2} \cos\left(\frac{\pi}{4} + x\right) = 0$. $x = \frac{\pi}{4} + 2k\pi$, $x = \frac{5\pi}{4} + 2k\pi, x = \frac{3\pi}{4} + 2k\pi, k \in \mathbf{Z}$.