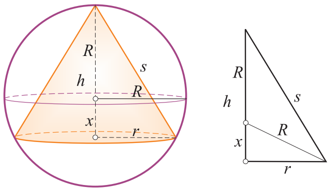
Zadatak 42. Zadanoj sferi upiši stožac s najvećom površinom plašta.

Rješenje.



Plašt stošca jednak je $P = r\pi s$. Iz trokuta sa slike slijedi sustav jednadžbi:

$$\begin{aligned}
&(R+x)^2 + r^2 = s^2\\
&x^2 + r^2 = R^2\\
\hline
&R^2 + 2Rx + x^2 - x^2 = s^2 - R^2\\
&2R^2 + 2Rx = s^2\\
&s^2 = 2Rh \qquad \left(\implies h = \frac{s^2}{2R}\right)\\
&s^2 = 2R\sqrt{s^2 - r^2}\\
&\frac{s^4}{4R^2} = s^2 - r^2\\
&r^2 = \frac{4R^2s^2 - s^4}{4R^2}\\
&r = \sqrt{\frac{4R^2s^2 - s^4}{4R^2}}.
\end{aligned}$$

Uvrstimo izraz za r u jednadžbu plašta:

$$\begin{aligned}
P &= \sqrt{\frac{(4R^2s^2 - s^4)}{4R^2}} \cdot \pi \cdot s\\
&= \frac{\pi}{2R}\sqrt{(4R^2s^2 - s^4)\cdot s^2}\\
&= \frac{\pi}{2R}\sqrt{4R^2s^4 - s^6}.
\end{aligned}$$

Deriviramo izraz za plašt i dobiveno izjednačimo s nulom:

$$\begin{aligned}
\frac{dP}{ds} &= \frac{\pi}{4R} \cdot \frac{16R^2s^3 - 6s^5}{2\sqrt{4R^2s^4 - s^6}}\\
&= \frac{\pi}{8R} \cdot \frac{2s^3(8R^2 - 3s^2)}{s^2\sqrt{4R^2 - s^2}}\\
&= \frac{s\pi}{4R} \cdot \frac{8R^2 - 3s^2}{\sqrt{4R^2 - s^2}}
\end{aligned}$$

$$\begin{aligned}
8R^2 - 3s^2 &= 0\\
s^2 &= \frac{8}{3}R^2\\
s &= \frac{2}{3}\sqrt{6}R,
\end{aligned}$$

odnosno $h = \dfrac{s^2}{2R} = \dfrac{\frac{8}{3}R^2}{2R} = \dfrac{4}{3}R$.