

Zadatak 14. Ako je $x_0 = 2^{-\log_4(4-2\sqrt{3})}$ te $f(x) = \frac{x^2 - 2x - 3}{x^2 + 4x + 3}$, izračunaj $f(x_0)$.

Rješenje.

Najprije,

$$f(x) = \frac{(x-3)(x+1)}{(x+3)(x+1)} = \frac{x-3}{x+3}, \quad x \neq -1;$$

a zatim

$$\begin{aligned} x_0 &= 2^{-\log_4(4-2\sqrt{3})} = 2^{-\log_2 2(\sqrt{3}-1)^2} = 2^{\log_2(\sqrt{3}-1)^{-1}} \\ &= \frac{1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{3}+1}{2}; \end{aligned}$$

$$f(x_0) = \frac{\frac{\sqrt{3}+1}{2} - 3}{\frac{\sqrt{3}+1}{2} + 3} = \frac{\sqrt{3}-5}{\sqrt{3}+7} \cdot \frac{7-\sqrt{3}}{7-\sqrt{3}} = \frac{12\sqrt{3}-38}{46} = \frac{6\sqrt{3}-19}{23}.$$