

Zadatak 15. Odredi $f(x)$, ako je:

$$1) f(\log_2 x) = \frac{\log_2(4x^3)}{\log_{\sqrt{2}} x - \log_{0.5} x};$$

$$2) f(\log_{\sqrt{3}} x) = \frac{\log_9 \sqrt{3x}}{\log_3 x - \log_{\frac{1}{3}} x};$$

$$3) f(\log_{\frac{1}{2}} x) = \frac{2 \log_4 x - \log_{\sqrt{2}} \frac{x}{8}}{1 + \log_2 \sqrt{0.125}};$$

$$4) f(\log_{0.2} x) = \frac{\log_{\sqrt{5}}(0.04x)}{\log_{25} 125 - \log_5 \sqrt{5x}}.$$

Rješenje. 1) $t = \log_2 x \iff 2^t = x$;

Sada dobivamo:

$$f(t) = \frac{\log_2(4 \cdot 2^{3t})}{\log_{\sqrt{2}} 2^t - \log_{0.5} 2^t} = \frac{\log_2(2^{3t+2})}{\log_2 2^{2t} + \log_2 2^t} = \frac{3t+2}{2t+t} = \frac{3t+2}{3t};$$

Dakle $f(x) = \frac{3x+2}{3x}$;

$$2) t = \log_{\sqrt{3}} x \iff (\sqrt{3})^t = x \iff x = 3^{t/2};$$

Sada dobivamo:

$$f(t) = \frac{\log_9 \sqrt{3 \cdot 3^{t/2}}}{\log_3 3^{t/2} - \log_{3^{-1}} 3^{t/2}} = \frac{\frac{1}{2} \log_3 3^{\frac{t+2}{4}}}{\log_3 3^{t/2} + \log_3 3^{t/2}} = \frac{\frac{t+2}{8}}{\frac{t}{2} + \frac{t}{2}} = \frac{t+2}{8t};$$

Dakle $f(x) = \frac{x+2}{8x}$;

$$3) t = \log_{\frac{1}{2}} x \iff \left(\frac{1}{2}\right)^t = x \iff x = 2^{-t};$$

Sada dobivamo:

$$\begin{aligned} f(t) &= \frac{2 \log_4 2^{-t} - \log_{\sqrt{2}} \frac{2^{-t}}{8}}{1 + \log_2 \sqrt{\frac{1}{8}}} = \frac{\log_2 2^{-t} - 2 \log_2 2^{-t-3}}{1 + \log_2 2^{-\frac{3}{2}}} \\ &= \frac{-t + 2(t+3)}{1 - \frac{3}{2}} = -2t - 12; \end{aligned}$$

Dakle $f(x) = -2x - 12$;

$$4) t = \log_{0.2} x \iff (0.2)^t = x \iff x = 5^{-t};$$

Sada dobivamo:

$$\begin{aligned}f(t) &= \frac{\log_{\sqrt{5}} \left(\frac{1}{25} \cdot 5^{-t} \right)}{\log_5 5^3 - \log_5 \sqrt{5 \cdot 5^{-t}}} = \frac{2 \log_5 5^{-t-2}}{\frac{3}{2} \log_5 5 - \log_5 5^{\frac{1-t}{2}}} = \frac{-2(t+2)}{\frac{3}{2} - \frac{1-t}{2}} \\&= \frac{-4(t+2)}{t+2} = -4;\end{aligned}$$

Dakle $f(x) = -4$.