

Zadatak 20. Izračunaj $\frac{f(x+h) - f(x)}{h}$ za sljedeće funkcije f :

1) $f(x) = 3x + 2$;

2) $f(x) = \frac{1}{3}x - \frac{2}{5}$;

3) $f(x) = x^2 - 2x + 3$;

4) $f(x) = x^3 - 3x + 4$;

5) $f(x) = x^4 - x^2 + 1$;

6) $f(x) = \frac{1}{x}$.

Rješenje.

1) $f(x) = 3x + 2$,

$$\frac{f(x+h) - f(x)}{h} = \frac{3x + 3h + 2 - 3x - 2}{h} = 3;$$

2) $f(x) = \frac{1}{3}x - \frac{2}{5}$,

$$\frac{f(x+h) - f(x)}{h} = \left(\frac{1}{3}x + \frac{1}{3}h - \frac{2}{5} - \frac{1}{3}x + \frac{2}{5} \right) \cdot \frac{1}{h} = \frac{1}{3};$$

3) $f(x) = x^2 - 2x + 3$,

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{x^2 + 2hx + h^2 - 2x - 2h + 3 - x^2 + 2x - 3}{h} \\ &= \frac{h^2 + 2hx - 2h}{h} = h + 2x - 2; \end{aligned}$$

4) $f(x) = x^3 - 3x + 4$,

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{x^3 + 3hx^2 + 3h^2x + h^3 - 3x - 3h + 4 - x^3 + 3x - 4}{h} \\ &= 3x^2 + 3hx + h^2 - 3; \end{aligned}$$

5) $f(x) = x^4 - x^2 + 1$,

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^2 - 2xh - h^2 + 1 - x^4 + x^2 - 1}{h} \\ &= 4x^3 + 6x^2h + 4xh^2 + h^3 - 2x - h = 4x^3 - 2x + h(6x^2 + 4xh + h^2 - 1); \end{aligned}$$

6) $f(x) = \frac{1}{x}$,

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x-x-h}{x(x+h)}}{h} = -\frac{1}{x(x+h)}.$$