Zadatak 22.   Funkcija $f : \mathbf{R} \to \mathbf{R}$ definirana s

$$f(x) = \begin{cases} 1 \text{ za racionalne } x, \\ 0 \text{ za iracionalne } x \end{cases}$$

zove se Dirichletova funkcija. Koliko je $f(-\pi)$, $f(\log_2 \sqrt[3]{4})$, $f(\sin 3)$, $f\left(\cos \dfrac{17\pi}{6}\right)$, $f(1.111\ldots)$, $f(4^{-0.5})$?

Rješenje. $f : \mathbf{R} \to \mathbf{R}$

$$f(x) = \begin{cases} 1, & x \in \mathbf{Q} \\ 0, & x \neq \mathbf{Q} \end{cases} \quad \text{– Dirichletova funkcija}$$

$f(-\pi) = 0;$

$f\left(\log_2 \sqrt[3]{4}\right) = f\left(\log_2 2^{\frac{2}{3}}\right) = f\left(\dfrac{2}{3}\right) = 1;$

$f(\sin 3) = f(0.052336) = 0;$

$f\left(\cos \dfrac{17\pi}{6}\right) = f\left(\cos \dfrac{5\pi}{6}\right) = f\left(-\dfrac{\sqrt{3}}{2}\right) = 0;$

$f(1.111\ldots) = f\left(\dfrac{10}{9}\right) = 1;$

$f(4^{-0.5}) = f\left(\dfrac{1}{\sqrt{4}}\right) = f\left(\dfrac{1}{2}\right) = 1.$