Zadatak 23. Odredi $f(x)$ ako je:

1) $f(x+1)=3x-2$; **2)** $f(x-\frac{1}{2})=-2x+\frac{1}{3}$;

3) $f(-\frac{2}{3}x+1)=x$; **4)** $f(2x-1)=4x^2-3$;

5) $f(x+3)=\frac{1}{2}x^2+2x+\frac{3}{2}$;

6) $f(3x-1)=x^2$.

Rješenje. **1)** $f(x+1)=3x-2$,

$$x+1=t \implies x=t-1 \implies f(t)=3(t-1)-2=3t-5$$
$$\implies f(x)=3x-5;$$

2) $f\left(x-\frac{1}{2}\right)=-2x+\frac{1}{3}$,

$$x-\frac{1}{2}=t \implies x=t+\frac{1}{2} \implies f(t)=-2t-1+\frac{1}{3}$$
$$\implies f(x)=-2x-\frac{2}{3};$$

3) $f\left(-\frac{2}{3}x+1\right)=x$,

$$-\frac{2}{3}x+1=t \implies -\frac{2}{3}x=t-1 \Big/ \cdot\left(-\frac{3}{2}\right)$$
$$\implies x=-\frac{3}{2}(t-1) \implies f(x)=-\frac{3}{2}(x-1);$$

4) $f(2x-1)=4x^2-3$,

$$2x-1=t \implies 2x=t+1 \implies x=\frac{t+1}{2}$$
$$f(t)=4\cdot\frac{t^2+2t+1}{4}-3 \implies f(x)=x^2+2x-2;$$

5) $f(x+3)=\frac{1}{2}x^2+2x+\frac{3}{2}$,

$$x+3=t \implies x=t-3;$$
$$f(t)=\frac{1}{2}(t-3)^2+2(t-3)+\frac{3}{2}=\frac{1}{2}(t^2-6t+9)+2(t-3)+\frac{3}{2}$$
$$=\frac{t^2}{2}-3t+\frac{9}{2}+2t-6+\frac{3}{2}=\frac{1}{2}t^2-t \implies f(x)=\frac{1}{2}x^2-x;$$

6) $f(3x-1)=x^2$,

$$3x-1=t \implies x=\frac{1}{3}(t+1);$$
$$f(t)=\frac{1}{9}(t+1)^2 \implies f(x)=\frac{1}{9}(x+1)^2.$$