

Zadatak 24. Odredi $f(x)$ ako je:

$$1) f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2};$$

$$2) f\left(1 + \frac{1}{x}\right) = x^2 - 1;$$

$$3) f\left(\frac{3x-1}{x+2}\right) = \frac{x+1}{x-1};$$

$$4) f\left(\frac{1}{x}\right) = \frac{3x}{2x+7};$$

$$5) f\left(\frac{1}{x^2}\right) = \frac{1-x^2}{1+x^2};$$

Rješenje.

$$1) f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2},$$

$$x + \frac{1}{x} = t \implies x^2 + \frac{1}{x^2} = t^2 - 2 \implies f(x) = x^2 - 2;$$

$$2) f\left(1 + \frac{1}{x}\right) = x^2 - 1,$$

$$1 + \frac{1}{x} = t \implies \frac{1}{x} = t - 1 \implies x = \frac{1}{t-1}$$

$$\implies x^2 = \frac{1}{(t-1)^2} \implies f(x) = \frac{1}{(x-1)^2} - 1, \quad x \neq 1;$$

$$3) f\left(\frac{3x-1}{x+2}\right) = \frac{x+1}{x-1},$$

$$\frac{3x-1}{x+2} = t \iff 3x-1 = tx+2t \iff (3-t)x = 2t+1 \iff x = \frac{2t+1}{3-t};$$

$$f(x) = \frac{\frac{2x+1}{3-x} + 1}{\frac{2x+1}{3-x} - 1} = \frac{2x+1+3-x}{2x+1-3+x} \implies f(x) = \frac{x+4}{3x-2}, \quad x \neq \frac{2}{3};$$

$$4) f\left(\frac{1}{x}\right) = \frac{3x}{2x+7} = \frac{3}{2+\frac{7}{x}} \implies f(x) = \frac{3}{7x+2}, \quad x \neq -\frac{2}{7};$$

$$5) f\left(\frac{1}{x^2}\right) = \frac{1-x^2}{1+x^2} = \frac{\frac{1}{x^2}-1}{\frac{1}{x^2}+1} \implies f(x) = \frac{x-1}{x+1}, \quad x > 0;$$

$$6) f\left(\frac{x}{x+1}\right) = x^2,$$

$$\frac{x}{x+1} = t \iff x = tx+t \iff (1-t)x = t \iff x = \frac{t}{1-t};$$

$$f(t) = \frac{t^2}{(1-t)^2} \implies f(x) = \left(\frac{x}{1-x}\right)^2, \quad x \neq 1.$$