Zadatak 41. Ako je $f\left(\alpha+\frac{3\pi}{2}\right)=\sin\alpha+\cos\alpha$, tada je $f\left(\frac{\pi}{2}-\alpha\right)\cdot f\left(\frac{\pi}{2}+\alpha\right)=\cos 2\alpha$. Dokaži!

Rješenje. $f\left(\alpha+\frac{3\pi}{2}\right)=\sin\alpha+\cos\alpha$,

$$\alpha+\frac{3\pi}{2}=t \iff \alpha=t-\frac{3\pi}{2} \implies f(t)=\sin\left(t-\frac{3\pi}{2}\right)+\cos\left(t-\frac{3\pi}{2}\right)=\cos t-\sin t;$$

$$f\left(\frac{\pi}{2}-\alpha\right)\cdot f\left(\frac{\pi}{2}+\alpha\right)=\cos\left[\left(\frac{\pi}{2}-\alpha\right)-\sin\left(\frac{\pi}{2}-\alpha\right)\right]\left[\cos\left(\frac{\pi}{2}+\alpha\right)-\sin\left(\frac{\pi}{2}+\alpha\right)\right]$$

$$=(\sin\alpha-\cos\alpha)(-\sin\alpha-\cos\alpha)=(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)$$

$$=\cos^2\alpha-\sin^2\alpha=\cos 2\alpha.$$