Zadatak 42. Odredi prirodno područje definicije funkcija:

1) $f(x) = \dfrac{1}{2-3x}$; **2)** $f(x) = \dfrac{1}{4x^2-1}$;

3) $f(x) = \dfrac{1}{\ln x}$; **4)** $f(x) = \dfrac{1}{\cos \pi x}$;

5) $f(x) = \sqrt{1-x^2}$; **6)** $f(x) = \sqrt{\dfrac{x}{x-1}}$;

7) $f(x) = \sqrt{\dfrac{2-x}{3x+2}}$; **8)** $f(x) = \sqrt{\log_{\frac{1}{2}} x}$;

9) $f(x) = \sqrt{3^{1-2x}}$; **10)** $f(x) = \sqrt{\sin \pi x}$.

Rješenje. **1)** $f(x) = \dfrac{1}{2-3x}$.

Nazivnik mora biti različit od nule: $2-3x \neq 0 \implies x \neq \dfrac{2}{3}$.

Dakle je $D_f = \mathbf{R} \setminus \left\{\dfrac{2}{3}\right\}$;

2) $f(x) = \dfrac{1}{4x^2-1}$.

Nazivnik mora biti različit od nule:

$4x^2-1 = (2x-1)(2x+1) \neq 0 \implies x \neq -\dfrac{1}{2}, \dfrac{1}{2}$.

Dakle je $D_f = \mathbf{R} \setminus \left\{-\dfrac{1}{2}, \dfrac{1}{2}\right\}$;

3) $f(x) = \dfrac{1}{\ln x}$.

Nazivnik mora biti različit od nule: $\ln x \neq 0 \implies x \neq 1$;

i argument logaritamske funkcije mora biti pozitivan: $x > 0$.

Dakle je $D_f = \mathbf{R}^+ \setminus \{1\}$;

4) $f(x) = \dfrac{1}{\cos \pi x}$.

Nazivnik mora biti različit od nule:

$\cos \pi x \neq 0 \implies \pi x \neq \dfrac{\pi}{2} + k\pi,\ k \in \mathbf{Z} \implies x \neq \dfrac{2k+1}{2},\ k \in \mathbf{Z}$.

Dakle je $D_f = \mathbf{R} \setminus \left\{x \in \mathbf{R} : x = \dfrac{2k+1}{2}, k \in \mathbf{Z}\right\}$;

5) $f(x) = \sqrt{1-x^2}$.

Izraz pod korijenom mora biti veći ili jednak nuli:

$1-x^2 \geqslant 0 \implies -1 \leqslant x \leqslant 1$.

Dakle je $D_f = [-1, 1]$;

6) $f(x) = \sqrt{\dfrac{x}{x-1}}$.

Izraz pod korijenom mora biti veći ili jednak nuli i nazivnik mu mora biti različit od nule:

$$\frac{x}{x-1} \geqslant 0 \implies \begin{array}{l} x \geqslant 0 \\ \underline{x-1>0} \\ x \in \langle 1, \infty \rangle \end{array} \quad \text{ili} \quad \begin{array}{l} x \leqslant 0 \\ \underline{x-1<0} \\ x \in \langle -\infty, 0] \end{array}$$

$$\implies D_f = \mathbf{R} \setminus \langle 0, 1] ;$$

7) $f(x) = \sqrt{\dfrac{2-x}{3x+2}}$

Izraz pod korijenom mora biti veći ili jednak nuli i nazivnik mu mora biti različit od nule:

$$\frac{2-x}{3x+2} \geqslant 0 \implies \begin{array}{l} 2-x \geqslant 0, \\ \underline{3x+2>0} \\ x \leqslant 2 \\ x > -\frac{2}{3} \\ \hline x \in \left\langle -\frac{2}{3}, 2 \right] \end{array} \quad \text{ili} \quad \begin{array}{l} 2-x \leqslant 0 \\ \underline{3x+2<0} \\ x \geqslant 2 \\ x < -\frac{2}{3} \\ \hline \emptyset \end{array}$$

$$\implies D_f = \left\langle -\frac{2}{3}, 2 \right];$$

8) $f(x) = \sqrt{\log_{\frac{1}{2}} x}$,

Uvjeti:

$x > 0$ (argument logaritamske funkcije mora biti pozitivan) i

$\log_{\frac{1}{2}} x \geqslant 0$ (izraz pod korijenom mora biti veći ili jednak nuli).

$$\begin{aligned} &\log_{\frac{1}{2}} x \geqslant 0 \\ &\log_{\frac{1}{2}} x \geqslant \log_{\frac{1}{2}} 1 \qquad \implies D_f = \langle 0, 1]; \\ &\qquad x \leqslant 1; \end{aligned}$$

9) $f(x) = \sqrt{3^{1-2x}}$, $3 > 0 \implies 3^{1-2x} > 0 \quad \forall x \in \mathbf{R} \implies D_f = \mathbf{R}$;

10) $f(x) = \sqrt{\sin \pi x}$;

$\sin \pi x \geqslant 0 \implies \pi x \in [2k\pi, \pi + 2k\pi], \quad k \in \mathbf{Z}, \quad x \in [2k, 2k+1], \quad k \in \mathbf{Z};$

$$\implies D_f = \bigcup_{x \in \mathbf{Z}} [2k, 2k+1] .$$