

Zadatak 43. Odredi prirodno područje definicije sljedećih funkcija

1) $f(x) = \sqrt{5 - x^2}$;

2) $f(x) = \log(6x - x^2)$;

3) $f(x) = \log x + \log(4 - x)$;

4) $f(x) = \frac{x^2 - 1}{x + 2}$;

5) $f(x) = \sqrt{\frac{x-1}{x+2}}$;

6) $f(x) = \sqrt{2 \sin x - 1}$.

Rješenje.

1) $f(x) = \sqrt{5 - x^2} \implies 5 - x^2 \geq 0$;

$5 - x^2 = 0$ je parabola s otvorom prema dole i nultočkama $-\sqrt{5}$, $\sqrt{5}$

$\implies D_f = [-\sqrt{5}, \sqrt{5}]$

2) $f(x) = \log(6x - x^2)$, $6x - x^2 > 0$, $x(6 - x) > 0$.

$x(6 - x) = 0$ je parabola s otvorom prema dole i nultočkama 0, 6

$\implies D_f = \langle 0, 6 \rangle$

3) $f(x) = \log x + \log(4 - x)$.

Uvjeti: $x > 0$, $x < 4 \implies D_f = \langle 0, 4 \rangle$

4) $f(x) = \frac{x^2 - 1}{x + 2}$, $x \neq -2 \implies D_f = \mathbf{R} \setminus \{-2\}$

5) $f(x) = \sqrt{\frac{x-1}{x+2}}$

Uvjeti: $\frac{x-1}{x+2} \geq 0$, $x \neq -2$;

$$\frac{x-1}{x+2} \geq 0 \implies \begin{array}{l} \underline{x-1 \geq 0} \\ \underline{x+2 > 0} \end{array} \implies \begin{array}{l} x \geq 1 \\ x > -2 \end{array} \implies x \in [1, \infty) \quad \text{ili} \quad \begin{array}{l} \underline{x-1 \leq 0} \\ \underline{x+2 < 0} \end{array} \implies \begin{array}{l} x \leq 1 \\ x < -2 \end{array} \implies x \in \langle -\infty, -2 \rangle \cup [1, \infty);$$

$D_f = \mathbf{R} \setminus [-2, 1)$

6) $f(x) = \sqrt{2 \sin x - 1}$

$2 \sin x - 1 \geq 0$, $\sin x \geq \frac{1}{2} \implies D_f = \bigcup_{k \in \mathbf{Z}} \left[\frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi \right]$