Zadatak 44. Odredi prirodno područje definicije sljedećih funkcija:

1) $f(x) = \log \dfrac{x+3}{x}$;

2) $f(x) = \sqrt{3+x} + \sqrt{3-x}$;

3) $f(x) = \sqrt{x^2+4x-5} \cdot \log_2(x+1)$;

4) $f(x) = \sqrt{4x-x^2} - \log_3(x-2)$;

5) $f(x) = \dfrac{\sqrt{x+5}}{\log(9-x)}$;

6) $f(x) = \log_{x-1}(x+1)$;

7) $f(x) = \log_{x+3}(x^2+1)$;

8) $f(x) = \sqrt{\log_{\frac{1}{3}}(x-1)}$;

9) $f(x) = \log_{\frac{2}{3}} \sin x$;

10) $f(x) = \sin\sqrt{x}$.

Rješenje. **1)** $f(x) = \log \dfrac{x+3}{x}$,

$$\frac{x+3}{x} > 0 \implies \begin{array}{r} x+3>0 \\ x>0 \\ \hline x>0; \end{array} \quad \text{ili} \quad \begin{array}{r} x+3<0 \\ x<0 \\ \hline x<-3; \end{array} \implies D_f = \mathbf{R} \setminus [-3,0]$$

2) $f(x) = \sqrt{3+x} + \sqrt{3-x}$,

$$\left.\begin{array}{l} 3+x \geqslant 0 \implies x \geqslant -3 \\ 3-x \geqslant 0 \implies x \leqslant 3 \end{array}\right\} \implies D_f = [-3,3]$$

3) $f(x) = \sqrt{x^2+4x-5} \cdot \log_2(x+1)$,

$$\left.\begin{array}{l} x^2+4x-5 \geqslant 0 \implies (x+5)(x-1) \geqslant 0 \\ \quad \implies x \in \langle -\infty, -5] \cup [1, +\infty\rangle \\ x+1>0 \implies x>-1 \end{array}\right\} \implies D_f = [1, +\infty\rangle$$

4) $f(x) = \sqrt{4x-x^2} - \log_3(x-2)$,

$$\left.\begin{array}{l} 4x-x^2 \geqslant 0 \implies x(4-x) \geqslant 0 \implies x \in [0,4] \\ x-2>0 \implies x>2 \end{array}\right\} \implies D_f = \langle 2, 4]$$

5) $f(x) = \dfrac{\sqrt{x+5}}{\log(9-x)}$,

$$\left.\begin{array}{l} x+5 \geqslant 0 \implies x \geqslant -5 \\ 9-x>0 \implies x<9 \\ 9-x \neq 1 \implies x \neq 8 \end{array}\right\} \implies D_f = [-5, 9\rangle \setminus \{8\}$$

6) $f(x) = \log_{x-1}(x+1)$,

$$\left.\begin{array}{l} x+1>0 \implies x>-1 \\ x-1>0 \implies x>1 \\ x-1\neq 1 \implies x\neq 2 \end{array}\right\} \implies D_f = \langle 1,+\infty\rangle \setminus \{2\}$$

7) $f(x) = \log_{x+3}(x^2+1)$,

$$\left.\begin{array}{l} x+3>0 \implies x>-3 \\ x+3\neq 1 \implies x\neq -2 \end{array}\right\} \implies D_f = \langle -3,+\infty\rangle \setminus \{-2\}$$

8) $f(x) = \sqrt{\log_{\frac{1}{3}}(x-1)}$,

$$\left.\begin{array}{l} x-1>0 \implies x>1 \\ \log_{\frac{1}{3}}(x-1)\geqslant 0 \implies 0<x-1\leqslant 1 \implies 1<x\leqslant 2 \end{array}\right\} \implies D_f = \langle 1,2]$$

9) $f(x) = \log_{\frac{2}{3}}\sin x$,

$$\sin x>0 \implies D_f = \bigcup_{k\in\mathbf{Z}} \langle 2k\pi,(2k+1)\pi\rangle$$

10) $f(x) = \sin\sqrt{x}$

$$x\geqslant 0 \implies D_f = \mathbf{R}_0^+$$