Zadatak 45. Odredi prirodno područje definicije sljedećih funkcija:

1) $f(x) = \sqrt{2+x-x^2}$;

2) $f(x) = \dfrac{1}{\sqrt{|x|-x}}$;

3) $f(x) = \dfrac{1}{\lfloor x \rfloor}$;

4) $f(x) = \dfrac{1}{\{x\}}$;

5) $f(x) = \sqrt{1-\lfloor x \rfloor}$;

6) $f(x) = \sqrt{\{x\}-1}$;

7) $f(x) = \sqrt{2^{x-1}-3^{x+1}}$;

8) $f(x) = \log_2(2-x)+\log_2(x+2)$;

9) $f(x) = \log_2 \log_{0.5} \dfrac{x+1}{x-2}$;

10) $f(x) = \sqrt{\log_{\frac{1}{2}} \dfrac{1-2x}{x+3}}$;

11) $f(x) = \sqrt{\log_x 2 - \log_2 x}$;

12) $f(x) = \sqrt{\log_{x-2}(x^2-8x+15)}$.

Rješenje. **1)** $f(x) = \sqrt{2+x-x^2}$,

$$2+x-x^2 \geqslant 0$$
$$x^2-x-2 \leqslant 0$$
$$(x-2)(x+1) \leqslant 0 \implies D_f = [-1,2]$$

2) $f(x) = \dfrac{1}{\sqrt{|x|-x}}$,

$$|x|-x > 0$$
$$x < |x| \implies D_f = \mathbf{R}^-$$

3) $f(x) = \dfrac{1}{\lfloor x \rfloor} \implies D_f = \mathbf{R} \setminus [0,1\rangle$

4) $f(x) = \dfrac{1}{\{x\}} \implies D_f = \mathbf{R} \setminus \mathbf{Z}$

5) $f(x) = \sqrt{1-\lfloor x \rfloor}$,

$$1-\lfloor x \rfloor \geqslant 0$$
$$\lfloor x \rfloor \leqslant 1 \implies D_f = \langle -\infty, 2\rangle$$

6) $f(x) = \sqrt{\{x\}-a} \implies D_f = \emptyset \qquad (\{x\} < 1)$

7) $f(x) = \sqrt{2^{x-1} - 3^{x+1}}$,

$$\begin{aligned}
2^{x-1} - 3^{x+1} &\geqslant 0 \\
2^{x-1} &\geqslant 3^{x+1} \Big/ : 3^{x-1} \\
\left(\frac{2}{3}\right)^{x-1} &\geqslant 9 \\
x - 1 &\leqslant \log_{\frac{2}{3}} 9 \\
x - 1 &\leqslant \frac{\log 9}{\log \frac{2}{3}} = \frac{2\log 3}{\log 2 - \log 3} \\
x &\leqslant \frac{2\log 3}{\log 2 - \log 3} + 1 \\
x &\leqslant \frac{2\log 3 + \log 2 - \log 3}{\log 2 - \log 3} = \frac{\log 6}{\log 2 - \log 3} \\
&\implies D_f = \left\langle -\infty, \frac{\log 6}{\log 2 - \log 3} \right]
\end{aligned}$$

8) $f(x) = \log_2(2-x) + \log_2(x+2) = \log_2(4-x^2)$, $x \neq \pm 2$

$$4 - x^2 \geqslant 0 \implies x^2 \leqslant 4 \implies D_f = \langle -2, 2 \rangle$$

9) $f(x) = \log_2 \log_{0.5} \dfrac{x+1}{x-2}$,

$$\frac{x+1}{x-2} > 0 \implies x \in \langle -\infty, -1 \rangle \cup \langle 2, +\infty \rangle$$

$$\log_{0.5} \frac{x+1}{x-2} > 0 \implies 0 < \frac{x+1}{x-2} \leqslant 1$$

$$\frac{x+1}{x-2} \leqslant 1 \implies \frac{x+1-x+2}{x-2} \leqslant 0 \implies \frac{3}{x-2} \leqslant 0 \implies x < 2$$

$$\implies D_f = \langle -\infty, -1 \rangle$$

10) $f(x) = \sqrt{\log_{\frac{1}{2}} \dfrac{1-2x}{x+3}}$,

$$\frac{1-2x}{x+3} > 0$$

$$\begin{array}{ccc}
1 - 2x > 0 & & 1 - 2x < 0 \\
x + 3 > 0 & & x + 3 < 0 \\
\hline
x < \frac{1}{2} & \text{ili} & x > \frac{1}{2} \\
x > -3 & & x < -3 \\
\hline
x \in \left\langle -3, \frac{1}{2} \right\rangle & & \emptyset
\end{array}$$

$$\implies x \in \left\langle -3, \frac{1}{2} \right\rangle$$

$$\log_{\frac{1}{2}} \frac{1-2x}{x+3} > 0 \implies 0 < \frac{1-2x}{x+3} \leqslant 1$$

$$\text{(i)} \quad \frac{1-2x}{x+3} > 0 \implies x \in \left\langle -3, \frac{1}{2} \right\rangle$$

$$\text{(ii)} \quad \frac{1-2x}{x+3} \leqslant 1 \implies \frac{1-2x-x-3}{x+3} \leqslant 0 \implies \frac{3x+2}{x+3} \geqslant 0$$

$$\frac{1-2x}{x+3} > 0$$

$$\begin{array}{c} 3x+2 \geqslant 0 \\ x+3 \geqslant 0 \\ \hline x \geqslant -\frac{2}{3} \\ x > -3 \\ \hline \end{array} \quad \text{ili} \quad \begin{array}{c} 3x+2 \leqslant 0 \\ x+3 \leqslant 0 \\ \hline x \leqslant -\frac{2}{3} \\ x < -3 \\ \hline \end{array}$$

$$\implies x \in \langle -\infty, -3\rangle \cup \left[-\frac{2}{3}, \infty \right\rangle$$

$$\implies D_f = \left[-\frac{2}{3}, \frac{1}{2} \right\rangle$$

11) $f(x) = \sqrt{\log_x 2 - \log_2 x}$, $x > 0$, $x \neq 1$,

$$\log_x 2 - \log_2 x \geqslant 0$$

$$\frac{1}{\log_2 x} - \log_2 x \geqslant 0$$

$$\frac{1 - \log_2^2 x}{\log_2 x} \geqslant 0$$

$$\left.\begin{array}{ll} \text{(i)} & -1 \leqslant \log_2 x \leqslant 1 \quad \text{i} \quad \log_2 x > 0 \\ & \frac{1}{2} \leqslant x \leqslant 2 \quad \text{i} \quad x > 1 \end{array}\right\} \implies x \in \langle 1, 2]$$

$$\left.\begin{array}{ll} \text{(ii)} & |\log_2 x| \geqslant 1 \quad \text{i} \quad \log x < 0 \\ & x \in \left\langle 0, \frac{1}{2} \right] \cup [2, +\infty\rangle \quad \text{i} \quad x \in \langle 0, 1\rangle \end{array}\right\} \implies x \in \left\langle 0, \frac{1}{2} \right]$$

$$\implies D_f = \left\langle 0, \frac{1}{2} \right] \cup \langle 1, 2]$$

12) $f(x) = \sqrt{\log_{x-2}(x^2 - 8x + 15)}$, $x \neq 3$,

$$x^2 - 8x + 15 > 0 \implies (x-3)(x-5) > 0 \implies x \in \langle -\infty, 3\rangle \cup \langle 5, +\infty\rangle$$

$$x - 2 > 0 \implies x > 2$$

$$\text{(i)} \quad x \in \langle 2, 3\rangle \implies 0 < x^2 - 8x + 15 \leqslant 1$$

$$x^2 - 8x + 14 \leqslant 0 \implies x \in [4 - \sqrt{2}, 3\rangle$$

$$\text{(ii)} \quad x > 3 \implies x^2 - 8x + 15 \geqslant 1$$

$$x^2 - 8x + 14 \geqslant 0 \implies x \in [4 + \sqrt{2}, +\infty\rangle$$

$$\implies D_f = [4 - \sqrt{2}, 3\rangle \cup [4 + \sqrt{2}, +\infty\rangle$$