Zadatak 46. Odredi prirodno područje definicije sljedećih funkcija:

1) $f(x) = \sqrt{\cos^2 x - \sin^2 x}$;

2) $f(x) = \sqrt{\cos(\sin x)}$;

3) $f(x) = \log_{\cos x}(\sin x)$;

4) $f(x) = \sqrt{\dfrac{1-\cos x}{1-\sin x}}$;

5) $f(x) = \sqrt{\dfrac{\cos x - 1}{\sin x}}$;

6) $f(x) = \sqrt{\sin^2 x - \sin x}$.

Rješenje. **1)** $f(x) = \sqrt{\cos^2 x - \sin^2 x}$,

$$\cos^2 x - \sin^2 x \geqslant 0 \implies \cos 2x \geqslant 0 \implies 2x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right], \quad k \in \mathbf{Z}$$

$$\implies D_f = \bigcup_{k\in\mathbf{Z}} \left[-\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi\right]$$

2) $f(x) = \sqrt{\cos(\sin x)}$,

$$\cos(\sin x) \geqslant 0$$

$$\sin x \in [-1, 1] \implies \cos(\sin x) > 0, \quad \forall x \in \mathbf{R} \implies D_f = \mathbf{R}$$

3) $f(x) = \log_{\cos x}(\sin x)$,

$$\sin x > 0 \implies x \in \bigcup_{k\in\mathbf{Z}} [2k\pi, (2k+1)\pi]$$

$$\cos x > 0 \implies x \in \bigcup_{k\in\mathbf{Z}} \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right]$$

$$\cos x \neq 1 \implies x \neq 2k\pi, k \in \mathbf{Z}$$

$$\implies D_f = \bigcup_{k\in\mathbf{Z}} \left\langle 2k\pi, \frac{\pi}{2} + 2k\pi\right]$$

4) $f(x) = \sqrt{\dfrac{1-\cos x}{1-\sin x}}$,

$$\frac{1-\cos x}{1-\sin x} \geqslant 0 \implies (1-\cos x)(1-\sin x) \geqslant 0, \quad \sin x \neq 1$$

$$\implies D_f = \mathbf{R} \setminus \bigcup_{k\in\mathbf{Z}} \left\langle \frac{\pi}{2} + 2k\pi \right\rangle$$

5) $f(x) = \sqrt{\dfrac{\cos x - 1}{\sin x}}$,

$$\sin x \neq 0 \implies x \neq k\pi, \quad k \in \mathbf{Z}$$

$$\frac{\cos x - 1}{\sin x} \geqslant 0 \implies \sin x < 0 \implies D_f = \bigcup_{k\in\mathbf{Z}} \langle\, (2k-1)\pi, 2k\pi \rangle$$

6) $f(x) = \sqrt{\sin^2 x - \sin x}$,

$\sin^2 x - \sin x \geqslant 0 \implies \sin x(\sin x - 1) \geqslant 0 \implies \sin x \leqslant 0 \quad \text{ili} \quad \sin x = 1$

$$\implies D_f = \bigcup_{k \in \mathbf{Z}} [(2k-1)\pi, 2k\pi] \cup \bigcup_{k \in \mathbf{Z}} \left\langle \frac{\pi}{2} + 2k\pi \right\rangle$$