Zadatak 47. Odredi prirodno područje definicije funkcije:

1) $f(x)=\sqrt{\log_2\left(1-\frac{1}{x}\right)}$;

2) $f(x)=\sqrt{1-\frac{1}{x^2}}$;

3) $f(x)=\sqrt{\log_{\frac{1}{2}}\frac{1}{|x|-1}}$;

4) $f(x)=\sqrt{\frac{\sin x+\cos x}{\sin x-\cos x}}$;

5) $f(x)=\frac{\log(x^2+1)}{\sqrt{1-4x^2}}$;

6) $f(x)=\frac{1}{\sqrt{\log_2(x^2-x-1)}}$;

7) $f(x)=\sqrt{9-\frac{4}{x^2}}$;

8) $f(x)=\sqrt{1-\frac{x+1}{x-1}}$;

9) $f(x)=\sqrt{\frac{1+\log_{\frac{1}{2}}x}{1+\log_{\sqrt{2}}x}}$;

10) $f(x)=\sqrt{\log_{\frac{1}{3}}(2x+1)+\log_3(x+1)}$.

Rješenje. **1)** $f(x)=\sqrt{\log_2\left(1-\frac{1}{x}\right)},\quad x\neq 0$;

$$1-\frac{1}{x}>0,\qquad \frac{x-1}{x}>0 \implies x>1 \text{ ili } x<0;$$
$$\log_2\left(1-\frac{1}{x}\right)\geqslant 0 \implies 1-\frac{1}{x}\geqslant 1,\qquad \frac{x-1-x}{x}\geqslant 0 \implies x<0;$$
$$\implies D_f=\langle-\infty,0\rangle\,.$$

2) $f(x)=\sqrt{1-\frac{1}{x^2}}$;

$$1-\frac{1}{x^2}\geqslant 0 \implies \frac{x^2-1}{x^2}\geqslant 0 \implies x^2-1\geqslant 0 \implies x\leqslant -1 \text{ ili } x\geqslant 1;$$

3) $f(x)=\sqrt{\log_{\frac{1}{2}}\frac{1}{|x|-1}}$;

$$|x|-1\neq 0 \implies x\neq -1,1;$$
$$\frac{1}{|x|-1}>0 \implies x\in\langle-\infty,-1\rangle\cup\langle 1,\infty\rangle\,;$$
$$\log_{\frac{1}{2}}\tfrac{1}{|x|-1}\geqslant 0 \implies 0<\frac{1}{|x|-1}\leqslant 1$$

$$\frac{1}{|x|-1} \leqslant 1, \quad \frac{2-|x|}{|x|-1} \leqslant 0 \implies$$

$$\begin{array}{lcl} 2-|x| \leqslant 0 & & 2-|x| \leqslant 0 \\ \underline{|x|-1>0} & & \underline{|x|-1<0} \\ |x| \geqslant 2 & \text{ili} & |x| \leqslant 2 \\ \underline{|x|>1} & & \underline{|x|<1} \\ x \in \langle -\infty, -2] \cup [2, \infty\rangle & & x \in \langle -1, 1\rangle \end{array}$$

$$\implies D_f = \langle -\infty, -2] \cup [2, \infty\rangle\,;$$

4) $f(x) = \sqrt{\dfrac{\sin x + \cos x}{\sin x - \cos x}}$;

$\sin x - \cos x \neq 0 \implies x \neq \dfrac{\pi}{4} + k\pi\,;$

$$\frac{\sin x + \cos x}{\sin x - \cos x} \geqslant 0 \implies (\sin x + \cos x)(\sin x - \cos x) \geqslant 0$$
$$\implies \sin^2 x - \cos^2 x \geqslant 0 \implies \cos 2x \leqslant 0$$
$$\implies 2x \in \left[\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi\right] \implies x \in \left[\frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi\right];$$

$$D_f = \left\langle \frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi\right].$$

5) $f(x) = \dfrac{\log(x^2+1)}{\sqrt{1-4x^2}}$;

$1 - 4x^2 > 0 \implies (1-2x)(1+2x) > 0 \implies x \in \left\langle -\dfrac{1}{2}, \dfrac{1}{2}\right\rangle;$

$$D_f = \left\langle -\frac{1}{2}, \frac{1}{2}\right\rangle;$$

6) $f(x) = \dfrac{1}{\sqrt{\log_2(x^2-x-1)}}$;

$x^2 - x - 1 > 0\,;$

$x^2 - x - 1 = 0, \quad x_{1,2} = \dfrac{1 \pm \sqrt{1+4}}{2} = \dfrac{1 \pm \sqrt{5}}{2} \implies$

$$x \in \left\langle -\infty, \frac{1-\sqrt{5}}{2}\right\rangle \cup \left\langle \frac{1+\sqrt{5}}{2}, \infty\right\rangle;$$

$$\log_2(x^2-x-1) > 0 \implies x^2 - x - 1 > 1 \implies x^2 - x - 2 > 0$$
$$\implies (x+1)(x-2) > 0 \implies x \in \langle -\infty, -1\rangle \cup \langle 2, \infty\rangle\,;$$

$$D_f = \langle -\infty, -1\rangle \cup \langle 2, \infty\rangle\,.$$

7) $f(x) = \sqrt{9 - \dfrac{4}{x^2}},\ x \neq 0\,;$

$$9-\frac{4}{x^2}\geqslant 0 \implies \frac{9x^2-4}{x^2}\geqslant 0 \implies 9x^2-4\geqslant 0 \implies (3x-2)(3x+2)\geqslant 0$$
$$\implies x\in\left\langle -\infty,-\frac{2}{3}\right]\cup\left[\frac{2}{3},\infty\right\rangle;$$
$$\implies D_f=\left\langle -\infty,-\frac{2}{3}\right]\cup\left[\frac{2}{3},\infty\right\rangle;$$

8) $f(x)=\sqrt{1-\frac{x+1}{x-1}}=\sqrt{\frac{x-1-x-1}{x-1}}=\sqrt{\frac{-2}{x-1}};$

$$x-1<0 \implies x<1 \implies D_f=\langle -\infty,1\rangle;$$

9) $f(x)=\sqrt{\frac{1+\log_{\frac{1}{2}}x}{1+\log_{\sqrt{2}}x}}=\sqrt{\frac{1-\log_2 x}{1+2\log_2 x}};$

$$\frac{1-\log_2 x}{1+2\log_2 x}\geqslant 0 \implies$$

$1-\log_2 x\geqslant 0$		$1-\log_2 x\leqslant 0$
$1+2\log_2 x>0$		$1+2\log_2 x<0$
$\log_2\leqslant 1$		$\log_2\geqslant 1$
$\log_2 x>-\frac{1}{2}$	ili	$\log_2 x<-\frac{1}{2}$
$x\leqslant 2$		$x\geqslant 2$
$x>\frac{\sqrt{2}}{2}$		$x<\frac{\sqrt{2}}{2}$
$x\in\left\langle\frac{\sqrt{2}}{2},2\right]$		$\emptyset$

$$\implies D_f=\left\langle\frac{\sqrt{2}}{2},2\right];$$

10) $f(x)=\sqrt{\log_{\frac{1}{3}}(2x+1)+\log_3(x+1)}=\sqrt{\log_3(x+1)-\log_3(2x+1)}$

$$=\sqrt{\log_3\frac{x+1}{2x+1}}$$

$$\frac{x+1}{2x+1}>0 \implies x>-\frac{1}{2} \text{ ili } x<-1 \implies x\in\langle -\infty,-1\rangle\cup\left\langle -\frac{1}{2},\infty\right\rangle;$$

$$\log_3\frac{x+1}{2x+1}\geqslant 0 \implies \frac{x+1}{2x+1}\geqslant 1 \implies \frac{-x}{2x+1}\geqslant 0 \implies$$

$-x\geqslant 0$		$-x\leqslant 0$
$2x+1>0$	ili	$2x+1<0$
$x\in\left\langle -\frac{1}{2},0\right]$		$\emptyset$

$$\implies D_f=\left\langle -\frac{1}{2},0\right].$$