Rješenja zadataka 3.2

Zadatak 1. Odredi kompozicije $f \circ g$, $g \circ f$, $f \circ f$ i $g \circ g$ za funkcije

1) $f(x) = 2x - 3$, $g(x) = x + 3$;

2) $f(x) = -\frac{1}{2}x + 1$; $g(x) = \frac{2}{3}x - \frac{5}{3}$;

3) $f(x) = 2x + 1$, $g(x) = x^2 - 1$.

Rješenje. **1)** $f(x) = 2x - 3$, $g(x) = x + 3$

$$(f \circ g)(x) = f[g(x)] = 2(x+3) - 3 = 2x + 3$$
$$(g \circ f)(x) = g[f(x)] = 2x - 3 + 3 = 2x$$
$$(f \circ f)(x) = f[f(x)] = 2(2x-3) - 3 = 4x - 9$$
$$(g \circ g)(x) = g[g(x)] = x + 3 + 3 = x + 6$$

2) $f(x) = \frac{1}{2}x + 1$, $g(x) = \frac{2}{3}x - \frac{5}{3}$

$$(f \circ g)(x) = -\frac{1}{2}\left(\frac{2}{3}x - \frac{5}{3}\right) + 1 = -\frac{1}{3}x + \frac{5}{6} + 1 = -\frac{1}{3}x + \frac{11}{6}$$
$$(g \circ f)(x) = \frac{2}{3}\left(-\frac{1}{2}x + 1\right) - \frac{5}{3} = -\frac{1}{3}x + \frac{2}{3} - \frac{5}{3} = -\frac{1}{3}x - 1$$
$$(f \circ f)(x) = -\frac{1}{2}\left(-\frac{1}{2}x + 1\right) + 1 = \frac{1}{4}x - \frac{1}{2} + 1 = \frac{1}{4}x + \frac{1}{2}$$
$$(g \circ g)(x) = \frac{2}{3}\left(\frac{2}{3}x - \frac{5}{3}\right) - \frac{5}{3} = \frac{4}{9}x - \frac{10}{9} - \frac{5}{3} = \frac{4}{9}x - \frac{25}{9}$$

3) $f(x) = 2x + 1$, $g(x) = x^2 - 1$

$$(f \circ g)(x) = 2(x^2 - 1) + 1 = 2x^2 - 2 + 1 = 2x^2 - 1$$
$$(g \circ f)(x) = (2x+1)^2 - 1 = 4x^2 + 4x + 1 - 1 = 4x^2 + 4x$$
$$(f \circ f)(x) = 2(2x+1) + 1 = 4x + 2 + 1 = 4x + 3$$
$$(g \circ g)(x) = (x^2 - 1)^2 - 1 = x^4 - 2x^2 + 1 - 1 = x^4 - 2x^2$$