

Zadatak 4. Ako je $f(x) = \log \frac{1+x}{1-x}$, $g(x) = \frac{3x+x^3}{3x^2+1}$, dokaži da je $(f \circ g)(x) = 3f(x)$.

Rješenje. $f(x) = \log \frac{1+x}{1-x}$, $g(x) = \frac{3x+x^3}{3x^2+1}$

$$\begin{aligned}(f \circ g)(x) &= \log \frac{1 + \frac{3x+x^3}{3x^2+1}}{1 - \frac{3x+x^3}{3x^2+1}} = \log \frac{3x^2+1+3x+x^3}{3x^2+1-3x-x^3} = \log \frac{1+3x+3x^2+x^3}{1-3x+3x^2-x^3} \\ &= \log \frac{(1+x)^3}{(1-x)^3} = \log \left(\frac{1+x}{1-x} \right)^3 = 3 \log \frac{1+x}{1-x} = 3f(x).\end{aligned}$$