

Zadatak 11. Odredi funkciju $f(x) = ax + b$ iz danog uvjeta:

1) $(f \circ f)(x) = f(x), \forall x \in \mathbf{R};$

2) $(f \circ f)(x) = x, \forall x \in \mathbf{R}.$

Rješenje.

$$f(x) = ax + b$$

1) $(f \circ f)(x) = f(x), \forall x \in \mathbf{R}$

$$a(ax + b) + b = ax + b$$

$$a^2x + ab + b = ax + b$$

$$a^2 = a \implies a = 0 \text{ ili } a = 1$$

$$(a + 1)b = b$$

$$a = 0 \implies b = b$$

$$a = 1 \implies b = 0$$

$$\implies f(x) = x \text{ ili } f(x) = b, b \in \mathbf{R};$$

2) $(f \circ f)(x) = x, \forall x \in \mathbf{R}$

$$a(ax + b) + b = x$$

$$a^2x + ab + b = x$$

$$\implies a^2 = 1 \implies a = \pm 1$$

$$(a + 1)b = 0$$

$$a = 1 \implies b = 0$$

$$a = -1 \implies b \in \mathbf{R}$$

$$\implies f(x) = x \text{ ili } f(x) = -x, b \in \mathbf{R};$$