

Zadatak 23. Odredi sva rješenja jednadžbe $(f \circ g)(x) = (g \circ f)(x)$ ako je:

- 1) $f(x) = x - 2$, $g(x) = x^2 - x + 3$;
- 2) $f(x) = 2x - 1$, $g(x) = 2x^2 - x + 1$;
- 3) $f(x) = 3x + 2$, $g(x) = x^2 + 2x - 3$.

Rješenje.

$$(f \circ g)(x) = (g \circ f)(x)$$

$$1) f(x) = x - 2, g(x) = x^2 - x + 3$$

$$(f \circ g)(x) = x^2 - x + 3 - 2 = x^2 - x + 1$$

$$(g \circ f)(x) = (x - 2)^2 - (x - 2) + 3 = x^2 - 4x + 4 - x + 2 + 3 = x^2 - 5x + 9$$

$$x^2 - 5x + 9 = x^2 - x + 1 \implies 4x = 8 \implies x = 2;$$

$$2) f(x) = 2x - 1, g(x) = 2x^2 - x + 1$$

$$(f \circ g)(x) = 2(2x^2 - x + 1) - 1 = 4x^2 - 2x + 2 - 1 = 4x^2 - 2x + 1$$

$$(g \circ f)(x) = 2(2x - 1)^2 - (2x - 1) + 1 = 8x^2 - 8x + 2 - 2x + 1 + 1$$

$$= 8x^2 - 10x + 4$$

$$4x^2 - 2x + 1 = 8x^2 - 10x + 4 \implies 4x^2 - 8x + 3 = 0$$

$$\implies 4x^2 - 2x - 6x + 3 = 0$$

$$\implies (2x - 3)(2x - 1) = 0 \implies x_1 = \frac{3}{2}, x_2 = \frac{1}{2};$$

$$3) f(x) = 3x + 2, g(x) = x^2 + 2x - 3$$

$$(f \circ g)(x) = 3(x^2 + 2x - 3) + 2 = 3x^2 + 6x - 7$$

$$(g \circ f)(x) = (3x + 2)^2 + 2(3x + 2) - 3 = 9x^2 + 12x + 4 + 6x + 4 - 3$$

$$= 9x^2 + 18x + 5$$

$$3x^2 + 6x - 7 = 9x^2 + 18x + 5 \implies 6x^2 + 12x + 12 = 0$$

$$\implies x^2 + 2x + 2 = 0$$

$$(x + 1)^2 = -1$$

$$x + 1 = \pm i, \quad x_{1,2} = -1 \pm i$$

$$D = 2^2 - 4 \cdot 2 = 4 - 8 = -4 < 0 \implies \text{nema realnih rješenja.}$$