Zadatak 30. Odredi kompoziciju $g \circ f$ funkcija $f(x) = ax - 1$ i $g(x) = x^2 + a$. Za koje je vrijednosti realnog parametra a zbroj recipročnih vrijednosti rješenja jednadžbe $(g \circ f)(x) = 0$ po apsolutnoj vrijednosti manji od 1?

Rješenje. $f(x) = ax - 1$, $g(x) = x^2 + a$, $a \in \mathbf{R}$

$$(g \circ f)(x) = (ax-1)^2 + a = a^2x^2 - 2ax + a + 1, \quad a^2x^2 - 2ax + a + 1 = 0$$

$$\Longrightarrow x_1 + x_2 = \frac{2}{a}; \qquad x_1x_2 = \frac{a+1}{a^2}, \qquad \left|\frac{1}{x_1} + \frac{1}{x_2}\right| < 1$$

$$\Longrightarrow \left|\frac{x_1 + x_2}{x_1x_2} < 1\right| < 1 \Longrightarrow \left|\frac{\frac{2}{a}}{\frac{a+1}{a^2}}\right| < 1 \Longrightarrow \left|\frac{2a}{a+1}\right| < 1$$

$$\Longrightarrow |2a| < |a+1| \Longrightarrow |2a| - |a+1| < 0$$

(i) $\begin{array}{c} a < -1 \\ -2a + a + 1 < 0 \\ -a < -1 \\ a > 0 \\ \emptyset \end{array}$

(ii) $\begin{array}{c} -1 \leqslant a < 0 \\ -2a - a - 1 < 0 \\ -3a < 1 \\ a > -\frac{1}{3} \end{array}$

$$\Longrightarrow a \in \left\langle -\frac{1}{3}, 1 \right\rangle$$

(iii) $\begin{array}{c} a \geqslant 0 \\ 2a - a - 1 < 0 \\ a < 1 \\ a \in [0, 1\rangle \end{array}$