

Zadatak 31. Odredi kompoziciju $f \circ g$ funkcija $f(x) = x^2 - x + 1$ i $g(x) = x + a$. Za koje je $a \in \mathbf{R}$ zbroj kubova korijena polinoma $f \circ g$ jednak nuli?

Rješenje. $f(x) = x^2 - x + 1$, $g(x) = x + a$, $a \in \mathbf{R}$

$$\begin{aligned}(f \circ g)(x) &= (x + a)^2 - (x + a) + 1 = x^2 + 2ax + a^2 - x - a + 1 \\ &= x^2 + (2a - 1)x + a^2 - a + 1\end{aligned}$$

$$\implies x_1 + x_2 = 1 - 2a, x_1x_2 = a^2 - a + 1$$

$$x_1^3 + x_2^3 = 0 \implies (x_1 + x_2)^3 - 3(x_1 + x_2)(x_1x_2) = 0$$

$$\implies (x_1 + x_2)[(x_1 + x_2)^2 - 3x_1x_2] = 0$$

$$\implies (1 - 2a)[1 - 4a + 4a^2 - 3(a^2 - a + 1)] = 0 \implies a_1 = \frac{1}{2}$$

$$\begin{aligned}4a^2 - 4a + 1 - 3a^2 + 3a - 3 = 0 &\implies a^2 - a - 2 = 0 \implies (a - 2)(a + 1) = 0 \\ &\implies a_2 = 2, a_3 = -1.\end{aligned}$$