Zadatak 33. Riješi jednadžbu $(f\circ g)(x)=(g\circ f)(x)$ ako je

1) $f(x)=10^{x-1}$, $g(x)=\log(2x)$;

2) $f(x)=5^{1-x}$, $g(x)=\log_{0.2}(2x)$.

Rješenje. **1)** $f(x)=10^{x-1}$, $g(x)=\log(2x)$

$$\begin{aligned}(f\circ g)(x)&=10^{\log(2x)-1}=10^{\log(2x)-\log 10}=10^{\vee\frac{2x}{10}}\\&=\frac{2x}{10}=\frac{x}{5}\\(g\circ f)(x)&=\log(2\cdot 10^{x-1})=\log 2+\log 10^{x-1}=x-1+\log 2\\&=x-\log 10+\log 2=x-\log 5\end{aligned}$$

$$\frac{x}{5}=x-\log 5\implies x=5x-5\log 5\implies 4x=5\log 5$$
$$\implies x=\frac{5}{4}\log 5.$$

2) $f(x)=5^{1-x}$, $g(x)=\log_{0.2}(2x)$

$$\begin{aligned}(f\circ g)(x)&=5^{1-\log_{0.2}(2x)}=5^{1+\log_5(2x)}=5^{\log_5 5+\log_5(2x)}\\&=5^{\log_5(10x)}=10x\\(g\circ f)(x)&=\log_{0.2}(2\cdot 5^{1-x})=-\log_5 2-\log_5 5^{1-x}=x-1-\log_5 2\end{aligned}$$

$$10x=x-1-\log_5 2$$

$$9x=-1-\log_5 2\implies x=-\frac{1}{9}(1+\log_5 2)<0$$

no $g(x):\mathbf{R}^+\to\mathbf{R}$, pa jednadžba nema rješenja.