

Zadatak 37.Odredi $a \in \mathbf{R}$ uz uvjet da jednadžba

$$(f \circ g)(x) = 1 - |x|, \text{ gdje je } f(x) = 2^{-x}, \\ g(x) = \log_{\frac{1}{2}} |x - a|, \text{ ima barem jedno rješenje.}$$

Rješenje.

$$f(x) = 2^{-x}, \quad g(x) = \log_{\frac{1}{2}} |x - a|, \quad a \in \mathbf{R}$$

$$(f \circ g)(x) = 2^{-\log_{\frac{1}{2}} |x-a|} = 2^{\log_2 |x-a|} = |x-a| \\ |x-a| = 1 - |x| \implies |x-a| + |x| = 1 \implies |x-a| \leq 1 \\ (|x| + |y| = 1 \implies |x| \leq 1, |y| \leq 1) \\ |x-a| \leq 1 \text{ i } |x-a| > 0 \implies |a| \leq 1.$$