

Zadatak 38. Za funkcije $f(x) = \log_{\frac{1}{5}} |x+a|$, $g(x) = 5^{1-x}$ odredi kompoziciju $g \circ f$. Za koje $a \in \mathbf{R}$ jednačba $(g \circ f)(x) = 1 - x$ ima rješenja?

Rješenje. $f(x) = \log_{\frac{1}{5}} |x+a|$, $g(x) = 5^{1-x}$

$$(g \circ f)(x) = 5^{1 - \log_{\frac{1}{5}} |x+a|} = 5^{\log_5 5 + \log_5 |x+a|} = 5|x+a|, a \in \mathbf{R}$$

$$\underbrace{5|x+a|}_{\geq 0} = \underbrace{1-x}_{\geq 0 \text{ za } x \leq 1}$$

(i) $x < -a$

$$-5x - 5a = 1 - x \implies 4x = -5a - 1 \implies x = \frac{-5a - 1}{4}$$

$$\frac{-5a - 1}{4} \leq 1 \implies -5a - 1 < 4 \implies -5a < 5 \implies a \geq -1$$

(ii) $x > -a$

$$5x + 5a = 1 - x \implies 6x = 1 - 5a \implies x = \frac{1 - 5a}{6}$$

$$\frac{1 - 5a}{6} \leq 1 \implies 1 - 5a \leq 6 \implies -5a \leq 5 \implies a \geq -1$$

$$\implies a \geq -1.$$