Zadatak 43. Ako je $f(x)=\cos\frac{x}{2}$, $g(x)=|2x-1|$, riješi jednadžbu $(g\circ f)(x)=1$. Koliko rješenja ova jednadžba ima na intervalu $[-10,10]$?

Rješenje. $f(x)=\cos\frac{x}{2}$, $g(x)=|2x-1|$

$$(g\circ f)(x)=\left|2\cos\frac{x}{2}-1\right|=1 \quad \Big/ : 2$$

$$\left|\cos\frac{x}{2}-\frac{1}{2}\right|=\frac{1}{2}$$

(i) $\cos\frac{x}{2}<\frac{1}{2} \implies \frac{x}{2}\in\bigcup_{k\in\mathbf{Z}}\left(\frac{\pi}{3}+2k\pi,\frac{5\pi}{3}+2k\pi\right) \implies x\in\bigcup_{k\in\mathbf{Z}}\left(\frac{2\pi}{3}+4k\pi,\frac{10\pi}{3}+4k\pi\right)$

$\frac{1}{2}-\cos\frac{x}{2}=\frac{1}{2} \implies \cos\frac{x}{2}=0 \implies \frac{x}{2}=k\frac{\pi}{2} \implies x=k\pi, k\in\mathbf{Z}$
$\implies x=(2k+1)\pi, k\in\mathbf{Z}$

(ii) $\cos\frac{x}{2}\geqslant\frac{1}{2} \implies \frac{x}{2}\in\bigcup_{k\in\mathbf{Z}}\left[-\frac{\pi}{3}+2k\pi,\frac{\pi}{3}+2k\pi\right] \implies x\in\bigcup_{k\in\mathbf{Z}}\left[-\frac{2\pi}{3}+4k\pi,\frac{2\pi}{3}+4k\pi\right]$

$\cos\frac{x}{2}-\frac{1}{2}=\frac{1}{2} \implies \cos\frac{x}{2}=1 \implies \frac{x}{2}=2k\pi, k\in\mathbf{Z} \implies x=4k\pi, k\in\mathbf{Z} \implies x=(2k+1)\pi$ ili $x=4k\pi$, $k\in\mathbf{Z}$.

Na intervalu $[-10,10]$ ima pet rješenja:

$$x_1=-3\pi,\quad x_2=-\pi,\quad x_3=0,\quad x_4=\pi,\quad x_5=3\pi.$$