

**Zadatak 44.** Riješi jednađbu  $(f \circ g)(x) = (g \circ f)(x)$  ako je

1)  $f(x) = \sin \pi x$ ,  $g(x) = x - 1$ ,

2)  $f(x) = \sin \pi x$ ,  $g(x) = 2x$ .

*Rješenje.*

1)  $f(x) = \sin \pi x$ ,  $g(x) = x - 1$

$$(f \circ g)(x) = \sin \pi(x - 1)$$

$$(g \circ f)(x) = \sin \pi x - 1$$

$$\sin \pi(x - 1) = \sin \pi x - 1$$

$$-\sin(\pi - \pi x) = \sin \pi x - 1$$

$$-\sin \pi x = \sin \pi x - 1$$

$$2 \sin \pi x = 1$$

$$\sin \pi x = \frac{1}{2} \implies \pi x = \frac{\pi}{2} \pm \frac{\pi}{3} + 2k\pi, k \in \mathbf{Z}$$

$$\implies x = \frac{1}{2} \pm \frac{1}{3} + 2k, k \in \mathbf{Z}, \quad x = \frac{(-1)^k}{6} + k, k \in \mathbf{Z}$$

2)  $f(x) = \sin \pi x$ ,  $g(x) = 2x$

$$(f \circ g)(x) = \sin 2\pi x$$

$$(g \circ f)(x) = 2 \sin \pi x$$

$$\sin 2\pi x - 2 \sin \pi x = 0$$

$$2 \sin \pi x \cos \pi x - 2 \sin \pi x = 0$$

$$2 \sin \pi x (\cos \pi x - 1) = 0$$

$$\sin \pi x = 0 \implies \pi x = k\pi, k \in \mathbf{Z} \implies x = k, k \in \mathbf{Z}; \cos \pi x = 1 \implies \pi x = 2k\pi, k \in \mathbf{Z} \implies x = 2k, k \in \mathbf{Z} \implies x \in \mathbf{Z}.$$