Zadatak 7. Odredi inverzne funkcije eksponencijalnih funkcija:

1) $f(x) = 2^{-x}$; **2)** $f(x) = 3^{x-1} + 2$;

3) $f(x) = 10^{2x} - 1$; **4)** $f(x) = 2^{1-\frac{x}{2}}$;

5) $f(x) = 5^{3-x}$; **6)** $f(x) = 3 \cdot 2^{1-x} + 1$.

Rješenje. **1)** $f(x) = 2^{-x}$

$$x = 2^{-y} \implies x = \frac{1}{2^y} \implies 2^y = \frac{1}{x} \implies f^{-1}(x) = -\log_2 x$$

2) $f(x) = 3^{x-1} + 2$

$$x = 3^{y-1} + 2 \implies 3^{y-1} = x - 2 \implies y - 1 = \log_3(x-2)$$

$$\implies f^{-1}(x) = 1 + \log_3(x-2)$$

3) $f(x) = 10^{2x} - 1$

$$x = 10^{2y} - 1 \implies 10^{2y} = x + 1 \implies 2y = \log(x+1)$$

$$\implies f^{-1}(x) = \frac{1}{2}\log(x+1)$$

4) $f(x) = 2^{1-\frac{x}{2}}$

$$x = 2^{1-\frac{y}{2}} \implies 1 - \frac{y}{2} = \log_2 x \implies \frac{y}{2} = 1 - \log_2 x$$

$$\implies f^{-1}(x) = 2 - 2\log_2 x$$

5) $f(x) = 5^{3-x}$

$$x = 5^{3-y} \implies 3 - y = \log_5 x \implies y = 3 - \log_5 x \implies f^{-1}(x) = 3 - \log_5 x$$

6) $f(x) = 3 \cdot 2^{1-x} + 1$

$$x = 3 \cdot 2^{1-y} + 1 \implies 3 \cdot 2^{1-y} = x - 1 \implies 2^{1-y} = \frac{x-1}{3}$$

$$\implies 1 - y = \log_2 \frac{x-1}{3} \implies y = 1 - \log_2 \frac{x-1}{3}$$

$$\implies y = \log_2 2 + \log_2 \frac{3}{x-1} \implies f^{-1}(x) = \log_2 \frac{6}{x-1}, x > 1$$