

Zadatak 9. Odredi inverzne funkcije sljedećih funkcija:

- 1) $f(x) = \log_3 x - \log_{\sqrt{3}} x;$
- 2) $f(x) = \log_2(4x) - \log_{\sqrt{2}} x;$
- 3) $f(x) = \log_{\sqrt{2}}(2x) + \log_{0.5} x;$
- 4) $f(x) = \log_{\frac{1}{2}}(4x) + \frac{1}{2} \log_{\sqrt{2}} x^2;$
- 5) $f(x) = \log_{\frac{1}{3}} \frac{x}{9} + \log_{3\sqrt{3}} x^3;$
- 6) $f(x) = \log_{\sqrt[3]{2}} \sqrt{x} + \log_{0.25} \frac{x}{4}.$

Rješenje.

1) $f(x) = \log_3 x - \log_{\sqrt{3}} x$

$$\begin{aligned} x &= \log_3 y - \log_{\sqrt{3}} y = \log_3 y - 2 \log_3 y = -\log_3 y = \log_3 \frac{1}{y} \\ \frac{1}{y} &= 3^x \implies f^{-1}(x) = 3^{-x} \end{aligned}$$

2) $f(x) = \log_2(4x) - \log_{\sqrt{2}} x$

$$\begin{aligned} x &= \log_2 4 + \log_2 y - 2 \log_2 y = \log_2 4 - \log_2 y = \log_2 \frac{4}{y} \\ \frac{4}{y} &= 2^x \implies \frac{1}{y} = 2^{x-2} \implies f^{-1}(x) = 2^{2-x} \end{aligned}$$

3) $f(x) = \log_{\sqrt{2}}(2x) + \log_{0.5} x$

$$\begin{aligned} x &= 2 \log_2 2 + 2 \log_2 y - \log_2 y = \log_2 4 + \log_2 y = \log_2(4y) \\ 4y &= 2^x \implies y = \frac{2^x}{4} \implies f^{-1}(x) = 2^{x-2} \end{aligned}$$

4) $f(x) = \log_{\frac{1}{2}}(4x) + \frac{1}{2} \log_{\sqrt{2}} x^2$

$$\begin{aligned} x &= -\log_2 4 - \log_2 y + 2 \log_2 y = -\log_2 4 + \log_2 y = \log_2 \frac{y}{4} \\ \frac{y}{4} &= 2^x \implies y = 4 \cdot 2^x \implies f^{-1}(x) = 2^{x+2} \end{aligned}$$

5) $f(x) = \log_{\frac{1}{3}} \frac{x}{9} + \log_{3\sqrt{3}} x^3$

$$\begin{aligned} x &= -\log_3 y + \log_3 9 + \frac{2}{3} \cdot 3 \cdot \log_3 y = \log_3 y + 2 \\ \log_3 y &= x - 2 \implies f^{-1}(x) = 3^{x-2} \end{aligned}$$

6) $f(x) = \log_{\sqrt[3]{2}} \sqrt{x} + \log_{0.25} \frac{x}{4}$

$$\begin{aligned} x &= 3 \cdot \frac{1}{2} \log_2 y - \frac{1}{2} \log_2 y + \frac{1}{2} \log_2 4 = \log_2 y + 1 \\ &\implies f^{-1}(x) = 2^{x-1} \end{aligned}$$