

Zadatak 10. Odredi inverznu funkciju funkcije

1) $f(x) = 2 \log_2(x - 1) + 1$;

2) $f(x) = \log_{\sqrt{2}} x - \log_{\frac{1}{2}} x - 1$;

3) $f(x) = \log_9 x + \log_{\frac{1}{3}} x$;

4) $f(x) = \log_2 \frac{x}{2} - \log_{\frac{1}{2}}(4x)$;

5) $f(x) = \frac{\log_3 x}{1 - \log_{\sqrt{3}} x}$;

6) $f(x) = 2 \left(\frac{1}{3}\right)^{-x} + 1$;

7) $f(x) = \log_{\sqrt{3}} x + \log_{\frac{1}{3}} \frac{x}{3}$;

8) $f(x) = \log_{\sqrt{2}} x - \log_{\frac{1}{2}} x - 1$.

Rješenje. 1) $f(x) = 2 \log_2(x - 1) + 1$;

$$\begin{aligned} x &= 2 \log_2(y - 1) + 1 \\ \frac{x - 1}{2} &= \log_2(y - 1) \\ 2^{\frac{x-1}{2}} + 1 &= y \\ f^{-1}(x) &= 2^{\frac{x-1}{2}} + 1; \end{aligned}$$

2) $f(x) = \log_{\sqrt{2}} x - \log_{\frac{1}{2}} x - 1$;

$$\begin{aligned} x &= \log_{\sqrt{2}} y - \log_{\frac{1}{2}} y - 1 \\ &= 2 \log_2 y + \log_2 y - 1 \\ \frac{x + 1}{3} &= \log_2 y \\ 2^{\frac{x+1}{3}} &= y \\ f^{-1}(x) &= 2^{\frac{x+1}{3}}; \end{aligned}$$

$$3) f(x) = \log_9 x + \log_{\frac{1}{3}} x;$$

$$x = \log_9 y + \log_{\frac{1}{3}} y$$

$$x = \frac{1}{2} \log_3 y - \log_3 y$$

$$x = -\frac{1}{2} \log_3 y$$

$$-2x = \log_3 y$$

$$3^{-2x} = y$$

$$f^{-1}(x) = 3^{-2x};$$

$$4) f(x) = \log_2 \frac{x}{2} - \log_{\frac{1}{2}}(4x);$$

$$x = \log_2 \frac{y}{2} - \log_{\frac{1}{2}}(4y)$$

$$x = \log_2 y - \log_2 2 + \log_2(4y)$$

$$x = \log_2 y - 1 + \log_2 4 + \log_2 y$$

$$x = 2 \log_2 y + 1$$

$$\frac{x-1}{2} = \log_2 y$$

$$2^{\frac{x-1}{2}} = y$$

$$f^{-1}(x) = 2^{\frac{x-1}{2}};$$

$$5) f(x) = \frac{\log_3 x}{1 - \log_{\sqrt{3}} x};$$

$$x = \frac{\log_3 y}{1 - \log_{\sqrt{3}} y}$$

$$x = \frac{\log_3 y}{1 - 2 \log_3 y}$$

$$x(1 - 2 \log_3 y) = \log_3 y$$

$$x - 2x \log_3 y = \log_3 y$$

$$x = \log_3 y(2x + 1)$$

$$\frac{x}{2x + 1} = \log_3 y$$

$$3^{\frac{x}{2x+1}} = y$$

$$f^{-1}(x) = 3^{\frac{x}{2x+1}};$$

$$6) f(x) = 2 \left(\frac{1}{3}\right)^{-x} + 1;$$

$$x = 2 \left(\frac{1}{3}\right)^{-y} + 1$$

$$x = 2 \cdot 3^y + 1$$

$$\frac{x-1}{2} = 3^y$$

$$\log_3 \frac{x-1}{2} = y$$

$$f^{-1}(x) = \log_3 \frac{x-1}{2};$$

$$7) f(x) = \log_{\sqrt{3}} x + \log_{\frac{1}{3}} \frac{x}{3};$$

$$x = \log_{\sqrt{3}} y + \log_{\frac{1}{3}} \frac{y}{3}$$

$$x = 2 \log_3 y - \log_3 \frac{y}{3}$$

$$x = 2 \log_3 y - \log_3 y + \log_3 3$$

$$x = \log_3 y + 1$$

$$y = 3^{x+1}$$

$$f^{-1}(x) = 3^{x+1};$$

$$8) f(x) = \log_{\sqrt{2}} x - \log_{\frac{1}{2}} x - 1.$$

$$x = \log_{\sqrt{2}} y - \log_{\frac{1}{2}} y - 1$$

$$x = 2 \log_2 y + \log_2 y - 1$$

$$x + 1 = 3 \log_2 y$$

$$\frac{x+1}{3} = \log_2 y$$

$$y = 2^{\frac{x+1}{3}}$$

$$f^{-1}(x) = 2^{\frac{x+1}{3}};$$