Zadatak 11. Odredi inverznu funkcije funkcije f za koju vrijedi

$$f\left(\log_{\frac{1}{3}} x\right) = \frac{\log_3 x}{1 - \log_{\sqrt{3}} x}.$$

Rješenje.

$$\begin{aligned}
f\left(\log_{\frac{1}{3}} x\right) &= \frac{\log_3 x}{1 - \log_{\sqrt{3}} x} \\
f\left(-\log_3 x\right) &= \frac{\log_3 x}{1 - 2\log_3 x} \\
t &= -\log_3 x \implies \\
f(t) &= \frac{-t}{1+2t} \implies f(x) = \frac{-x}{1+2x} \\
x &= \frac{-y}{1+2y} \\
x(1+2y) &= -y \\
x + 2xy &= -y \\
y + 2xy &= -x \\
y(1+2x) &= -x \\
y &= -\frac{x}{1+2x} \\
f^{-1}(x) &= -\frac{x}{1+2x}.
\end{aligned}$$