Zadatak 14. Ako je $f(x) = \dfrac{ax+1}{x+a}$, te $f(-1)+f(2)=4$, koliko je $f^{-1}(-1)+f^{-1}(2)$?

Rješenje. $f(-1)+f(2) = \dfrac{-a+1}{-1+a} + \dfrac{2a+1}{2+a} = -1 + \dfrac{2a+1}{2+a}4$;

$f(-1)+f(2)=4 \implies$

$$-1+\frac{2a+1}{2+a}=4$$
$$\frac{2a+1}{2+a}-5=0$$
$$\frac{2a+1-10-5a}{2+a}=0$$
$$\frac{-3a-9}{2+a}=0$$
$$\implies -3a-9=0 \implies a=-3;$$
$$\implies f(x)=\frac{-3x+1}{x-3},\ x\neq 3$$

$x = \dfrac{-3y+1}{y-3} \implies x(y-3) = -3y+1 \implies xy-3x = -3y+1 \implies$

$xy+3y = 3x+1 \implies y = \dfrac{3x+1}{x+3} \implies f^{-1}(x) = \dfrac{3x+1}{x+3},\ x\neq -3$

$f^{-1}(-1)+f^{-1}(2) = \dfrac{-3+1}{-1+3} + \dfrac{6+1}{2+3} = -1+\dfrac{7}{5} = \dfrac{2}{5}.$