Rješenja zadataka 3.4

Zadatak 1. Odredi intervale monotonosti za sljedeće funkcije:

1) $f(x) = -x^2 + 2x + 3$;

2) $f(x) = -(x-2)^3$; **3)** $f(x) = \sin 2x$;

4) $f(x) = \sin x + \cos x$;

5) $f(x) = -\dfrac{1}{x}$; **6)** $f(x) = \dfrac{1-x}{x}$.

Rješenje. **1)** $f(x) = -x^2 + 2x + 3 = -(x^2 - 2x + 1 - 1) + 3 = -(x-1)^2 + 4$

x	$\langle -\infty, 1\rangle$	$\langle 1, +\infty\rangle$
$f(x)$	$\nearrow$	$\searrow$

2) $f(x) = -(x-2)^3$

x	$\langle -\infty, +\infty\rangle$
$f(x)$	$\searrow$

3) $f(x) = \sin 2x$

$2x$	$\left\langle -\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi \right\rangle$	$\left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle$
$f(x)$	$\nearrow$	$\searrow$

, $k \in \mathbf{Z}$

$\Longrightarrow$

x	$\left\langle -\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi \right\rangle$	$\left\langle \frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi \right\rangle$
$f(x)$	$\nearrow$	$\searrow$

, $k \in \mathbf{Z}$

4) $f(x) = \sin x + \cos x = \sqrt{2}\left(\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x\right) = \sqrt{2}\left(\sin x \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos x\right)$

$= \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

$x + \frac{\pi}{4}$	$\left\langle -\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi \right\rangle$	$\left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle$
$\sin\left(x + \frac{\pi}{4}\right)$	$\nearrow$	$\searrow$

$\Longrightarrow$

x	$\left\langle -\frac{3\pi}{4} + 2k\pi, \frac{\pi}{4} + 2k\pi \right\rangle$	$\left\langle \frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi \right\rangle$
$f(x)$	$\nearrow$	$\searrow$

, $k \in \mathbf{Z}$

5) $f(x) = -\dfrac{1}{x}$

x	$\langle -\infty, 0\rangle$	$\langle 0, +\infty\rangle$
$f(x)$	$\nearrow$	$\nearrow$

, $x \neq 0$

6) $f(x)=\frac{1-x}{x}=\frac{1}{x}-1$

$$\begin{array}{c|cc} x & \langle -\infty, 0\rangle & \langle 0, +\infty\rangle \\ \hline f(x) & \nearrow & \nearrow \end{array}\quad , \; x \neq 0$$