Zadatak 4. Koje su od danih funkcija parne, a koje neparne: **1)** $f(x) = 2x^4 - x^2 + 13$; **2)** $f(x) = x^3 + 2x - 1$;

3) $f(x) = \sqrt{x^2 - 3}$; **4)** $f(x) = \sqrt[3]{x - x^3}$;

5) $f(x) = \dfrac{1}{4x^2 - 3}$; **6)** $f(x) = \dfrac{x}{x^2 - 1}$;

7) $f(x) = x^3 \cos x$; **8)** $f(x) = 5x \cdot \sin 2x$;

9) $f(x) = \sin^2 x$; **10)** $f(x) = \operatorname{tg} x^3$;

11) $f(x) = \dfrac{a^{2x} - 1}{a^x}$; **12)** $f(x) = \ln \dfrac{x+3}{x-3}$?

Rješenje. **1)** $f(x) = 2x^4 - x^2 + 13$

$$f(-x) = 2(-x)^4 - (-x)^2 + 13 = 2x^4 - x^2 + 13 = f(x) \implies \text{parna;}$$

2) $f(x) = x^3 + 2x - 1$

$$f(-x) = (-x)^3 + 2(-x) - 1 = -x^3 - 2x - 1 \implies \text{ni parna ni neparna;}$$

3) $f(x) = \sqrt{x^2 - 3}$

$$f(-x) = \sqrt{(-x)^2 - 3} = \sqrt{x^2 - 3} = f(x) \implies \text{parna;}$$

4) $f(x) = \sqrt[3]{x - x^3}$

$$f(-x) = \sqrt[3]{-x - (-x)^3} = \sqrt[3]{-x + x^3} = -\sqrt[3]{x - x^3} = -f(x) \implies \text{neparna;}$$

5) $f(x) = \dfrac{1}{4x^2 - 3}$

$$f(-x) = \frac{1}{4(-x)^2 - 3} = \frac{1}{4x^2 - 3} = f(x) \implies \text{parna;}$$

6) $f(x) = \dfrac{x}{x^2 - 1}$

$$f(-x) = \frac{-x}{(-x)^2 - 1} = \frac{-x}{x^2 - 1} = -f(x) \implies \text{neparna;}$$

7) $f(x) = x^3 \cos x$

$$f(-x) = (-x)^3 \cos(-x) = -x^3 \cos x = -f(x) \implies \text{neparna;}$$

8) $f(x) = 5x \sin 2x$

$$f(-x) = 5(-x) \sin 2(-x) = -5x(-\sin 2x) = 5x \sin 2x = f(x) \implies \text{parna;}$$

9) $f(x) = \sin^2 x$

$$f(-x) = \sin^2(-x) = \sin^2 x = f(x) \implies \text{parna;}$$

10) $f(x) = \operatorname{tg} x^3$

$$f(-x) = \operatorname{tg}(-x)^3 = \operatorname{tg}(-x^3) = -\operatorname{tg} x^3 = -f(x) \implies \text{neparna;}$$

11) $f(x) = \dfrac{a^{2x} - 1}{a^x}$

$$f(-x) = \frac{a^{-2x} - 1}{a^{-x}} = \frac{\frac{1}{a^{2x}} - 1}{\frac{1}{a^x}} = \frac{\frac{1 - a^{2x}}{a^{2x}}}{\frac{1}{a^x}} = \frac{1 - a^{2x}}{a^x} = -\frac{a^{2x} - 1}{a^x} = -f(x) \implies \text{neparna;}$$

12) $f(x) = \ln \dfrac{x+3}{x-3}$, $\dfrac{x+3}{x-3} > 0 \implies x \notin [-3, 3]$

$$f(-x) = \ln \frac{-x+3}{-x-3} = \ln \frac{-(x-3)}{-(x+3)} = \ln \left(\frac{x+3}{x-3}\right)^{-1} = -\ln \frac{x+3}{x-3} = -f(x) \implies \text{neparna.}$$