Zadatak 5. Koje su od sljedećih funkcija parne, koje su neparne, a koje nisu ni parne, ni neparne:

1) $f(x) = \log_2 x^2$;

2) $f(x) = |x-1|+|x+1|$;

3) $f(x) = \dfrac{x-1}{|x-1|}$;

4) $f(x) = \dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x}}}$;

5) $f(x) = \log_{\frac{1}{3}}(x+\sqrt{1+x^2})$;

6) $f(x) = \ln\dfrac{1-x}{1+x}$;

7) $f(x) = \sqrt{1-x+x^2}-\sqrt{1+x+x^2}$;

8) $f(x) = \left(x-\log_3\dfrac{2+x}{2-x}\right)\cdot\log_2\dfrac{x+1}{x-1}$;

9) $f(x) = \sin(\cos x)$;

10) $f(x) = \sin^2 x - \cos^2 x$;

11) $f(x) = \sin x + \cos x$;

12) $f(x) = \sin x \cdot \cos x$;

13) $f(x) = \ln\dfrac{\cos(x-1)}{\cos(x+1)}$;

14) $f(x) = \log_2\dfrac{1+\sin x}{1-\sin x}$?

Rješenje. **1)** $f(x) = \log_2 x^2$, $D_f = \mathbf{R}^*$

$$f(-x) = \log_2(-x)^2 = \log_2 x^2 = f(x) \implies \text{parna};$$

2) $f(x) = |x-1|+|x+1|$

$$f(-x) = |-x-1|+|-x+1| = |x+1|+|x-1| = f(x) \implies \text{parna};$$

3) $f(x) = \dfrac{x-1}{|x-1|}$, $x \neq 1$

$$f(-x) = \frac{-x-1}{|-x-1|} = \frac{-x-1}{|x+1|} \implies \text{ni parna ni neparna};$$

4) $f(x) = \dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x}}} = \dfrac{1}{x+\dfrac{1}{\dfrac{x^2+1}{x}}} = \dfrac{1}{x+\dfrac{x}{x^2+1}} = \dfrac{1}{\dfrac{x^3+x+x}{x^2+1}} = \dfrac{x^2+1}{x^3+2x}$

$$f(-x) = \frac{(-x)^2+1}{(-x)^3+2(-x)} = \frac{x^2+1}{-x^3-2x} = -\frac{x^2+1}{x^3+2x} = -f(x) \implies \text{neparna};$$

5) $f(x) = \log_{\frac{1}{3}}(x + \sqrt{1+x^2})$

$$f(-x) = \log_{\frac{1}{3}}(-x + \sqrt{1+(-x)^2}) = \log_{\frac{1}{3}}(-x + \sqrt{1+x^2}) \cdot \frac{x+\sqrt{1+x^2}}{x+\sqrt{1+x^2}}$$

$$= \log_{\frac{1}{3}} \frac{1+x^2-x^2}{x+\sqrt{1+x^2}} = -\log_{\frac{1}{3}}(x+\sqrt{1+x^2}) = -f(x) \implies \text{neparna;}$$

6) $f(x) = \ln\dfrac{1-x}{1+x}$, $D_f = \langle -1, 1\rangle$

$$f(-x) = \ln\frac{1+x}{1-x} = -\ln\frac{1-x}{1+x} = -f(x) \implies \text{neparna;}$$

7) $f(x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2}$

$$f(-x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2} = -f(x) \implies \text{neparna;}$$

8) $f(x) = \left(x - \log_3\dfrac{2+x}{2-x}\right)\cdot \log_2\dfrac{x+1}{x-1}$

$$f(-x) = \left(-x - \log_3\frac{2-x}{2+x}\right)\log_2\frac{-x+1}{-x-1} = \left(-x + \log_3\frac{2+x}{2-x}\right)\log_2\frac{x-1}{x+1}$$

$$= -\left(x - \log_3\frac{2+x}{2-x}\right)\cdot\left(-\log_2\frac{x+1}{x-1}\right) = f(x) \implies \text{parna;}$$

9) $f(x) = \sin(\cos x)$

$$f(-x) = \sin(\cos(-x)) = \sin(\cos x) = f(x) \implies \text{parna;}$$

10) $f(x) = \sin^2 x - \cos^2 x = -\cos 2x$

$$f(-x) = -\cos(-2x) = -\cos 2x = f(x) \implies \text{parna;}$$

11) $f(x) = \sin x + \cos x$

$$f(-x) = \sin(-x) + \cos(-x) = -\sin x + \cos x \implies \text{ni parna ni neparna;}$$

12) $f(x) = \sin x\cos x = \dfrac{1}{2}\sin 2x$

$$f(-x) = \frac{1}{2}\sin(-2x) = -\frac{1}{2}\sin 2x = -f(x) \implies \text{neparna;}$$

13) $f(x) = \ln\dfrac{\cos(x-1)}{\cos(x+1)}$

$$f(-x) = \ln\frac{\cos(-x-1)}{\cos(-x+1)} = \ln\frac{\cos(x+1)}{\cos(x-1)} = -\ln\frac{\cos(x-1)}{\cos(x+1)} = -f(x) \implies \text{neparna;}$$

14) $f(x) = \log_2\dfrac{1+\sin x}{1-\sin x}$

$$f(-x) = \log_2\frac{1+\sin(-x)}{1-\sin(-x)} = \log_2\frac{1-\sin x}{1+\sin x} = -\log_2\frac{1+\sin x}{1-\sin x} = -f(x) \implies \text{neparna;}$$