Zadatak 6. Je li funkcija f parna ili neparna:

1) $f(x) = \ln \dfrac{\sin x - \cos x}{\sin x + \cos x}$;

2) $f(x) = \dfrac{\operatorname{tg} x - \operatorname{ctg} x}{\operatorname{tg} x + \operatorname{ctg} x}$;

3) $f(x) = \ln \sqrt{\dfrac{1-x}{1+x}}$;

4) $f(x) = \sqrt{\dfrac{e^x - 1}{e^x + 1}}$;

Rješenje. **1)** $f(x) = \ln \dfrac{\sin x - \cos x}{\sin x + \cos x}$;

$f(-x) = \ln \dfrac{\sin(-x) - \cos(-x)}{\sin(-x) + \cos(-x)} = \ln \dfrac{-\sin x - \cos x}{-\sin x + \cos x} = \ln \dfrac{\sin x + \cos x}{\sin x - \cos x} \implies$ ni parna ni neparna;

2) $f(x) = \dfrac{\operatorname{tg} x - \operatorname{ctg} x}{\operatorname{tg} x + \operatorname{ctg} x}$;

$f(-x) = \dfrac{\operatorname{tg}(-x) - \operatorname{ctg}(-x)}{\operatorname{tg}(-x) + \operatorname{ctg}(-x)} = \dfrac{-\operatorname{tg} x + \operatorname{ctg} x}{-\operatorname{tg} x - \operatorname{ctg} x} = \dfrac{\operatorname{tg} x - \operatorname{ctg} x}{\operatorname{tg} x + \operatorname{ctg} x} = f(x) \implies$ parna;

3) $f(x) = \ln \sqrt{\dfrac{1-x}{1+x}}$;

$f(-x) = \ln \sqrt{\dfrac{1+x}{1-x}} \implies$ ni parna ni neparna;

4) $f(x) = \sqrt{\dfrac{e^x - 1}{e^x + 1}}$.

$f(-x) = \sqrt{\dfrac{e^{-x} - 1}{e^{-x} + 1}} = \sqrt{\dfrac{\frac{1}{e^x} - 1}{\frac{1}{e^x} + 1}} = \sqrt{\dfrac{1 - e^x}{1 + e^x}} \implies$ ni parna ni neparna.