Zadatak 12. Odredi temeljni period svake od sljedećih funkcija:

1) $f(x) = \frac{1}{2}\sin\frac{3}{2}x$;
2) $f(x) = 2\cos(2x-1)$;
3) $f(x) = \sin\frac{\pi}{3}x$;
4) $f(x) = \operatorname{tg}\frac{3\pi}{4}x$;
5) $f(x) = \operatorname{ctg}(2x-\frac{\pi}{6})$;
6) $f(x) = \operatorname{tg}6x$;
7) $f(x) = 2\sin x + \cos x$;
8) $f(x) = \sin\frac{x}{3} + \cos x$;
9) $f(x) = \sin\frac{3x}{2} - \cos\frac{x}{3}$;
10) $f(x) = \cos 3\pi x + \sin 2\pi x$;
11) $f(x) = |\sin x|$;
12) $f(x) = \cos^2 x$.

Rješenje. **1)** $f(x) = \frac{1}{2}\sin\frac{3}{2}x$

$$f(x+P) = \frac{1}{2}\sin\left(\frac{3}{2}x+\frac{3}{2}P\right) = \frac{1}{2}\sin\frac{3}{2}x$$
$$\frac{3}{2}P = 2\pi \implies P = \frac{4}{3}\pi;$$

2) $f(x) = 2\cos(2x-1)$

$$f(x) = a\underset{(\cos)}{\sin}(bx+c), \qquad P = \frac{2\pi}{|b|}$$

$$f(x+P) = 2\cos(2x-1+2P)$$
$$2P = 2\pi \implies P = \pi;$$

3) $f(x) = \sin\frac{\pi}{3}x$

$$P = \frac{2\pi}{\frac{\pi}{3}} \implies P = 6;$$

4) $f(x) = \operatorname{tg}\frac{3\pi}{4}x$

$$f(x) = a\underset{(\operatorname{ctg})}{\operatorname{tg}}(bx+c), \qquad P = \frac{\pi}{|b|}$$

$$P = \frac{\pi}{\frac{3\pi}{4}} \implies P = \frac{4}{3};$$

5) $f(x) = \text{ctg}\left(2x - \frac{\pi}{6}\right) \implies P = \frac{\pi}{2}$;

6) $f(x) = \text{tg}\, 6x \implies P = \frac{\pi}{6}$;

7) $f(x) = 2\sin x + \cos x$

$$P_1 = 2\pi, P_2 = 2\pi \implies P = 2\pi;$$

8) $f(x) = \sin\frac{x}{3} + \cos x$

$$P_1 = 6\pi, P_2 = 2\pi \implies P = 6\pi;$$

9) $f(x) = \sin\frac{3x}{2} - \cos\frac{x}{3}$

$$P_1 = \frac{2\pi}{\frac{3}{2}} = \frac{4}{3}\pi, P_2 = \frac{2\pi}{\frac{1}{3}} = 6\pi \implies P = 12\pi;$$

10) $f(x) = \cos 3\pi x + \sin 2\pi x$

$$P_1 = \frac{2\pi}{3\pi} = \frac{2}{3}, P_2 = \frac{2\pi}{2\pi} = 1 \implies P = 2;$$

11) $f(x) = |\sin x| \implies P = \pi$;

12) $f(x) = \cos^2 x$

$$\cos^2(x + P) = \cos^2 x$$

$$x = 0 \implies \cos^2 P = 1 \implies |\cos P| = 1 \implies P = \pi.$$