Zadatak 20. Grafički prikaži funkcije:

1) $f(x) = |\sin x| - \sin|x|$;

2) $f(x) = |\cos x| \cdot \operatorname{tg}|x|$;

3) $f(x) = \dfrac{\sin|2x|}{|\sin x|}$;

4) $f(x) = \dfrac{\sin x + \sin 3x}{\sqrt{2\cos 2x + 2}}$;

5) $f(x) = |\cos x - \sqrt{3}\sin x|$;

6) $f(x) = |\sin 2x + \sqrt{3}\cos 2x|$.

Rješenje. **1)**

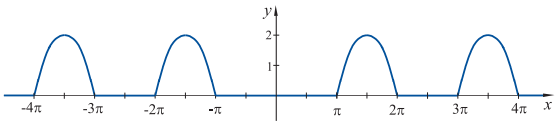
$$|\sin x| = \sin x \ \text{ za } \ x \in [2k\pi, \pi + 2k\pi],$$
$$|\sin x| = -\sin x \ \text{ za } \ x \in \langle \pi + 2k\pi, 2\pi + 2k\pi \rangle,$$
$$\sin|x| = \sin x \ \text{ za } \ x \in [0, \infty\rangle,$$
$$\sin|x| = -\sin x \ \text{ za } \ x \in \langle -\infty, 0\rangle;$$

$\underline{x \in [0, \infty\rangle}$:

$$f(x) = 2\sin x \ \text{ za } x \in [2k\pi, \pi + 2k\pi],$$
$$f(x) = 0 \ \text{ za } x \in \langle \pi + 2k\pi, 2\pi + 2k\pi \rangle;$$

$\underline{x \in \langle -\infty, 0\rangle}$:

$$f(x) = 0 \ \text{ za } x \in [2k\pi, \pi + 2k\pi],$$
$$f(x) = 2\sin x \ \text{ za } x \in \langle \pi + 2k\pi, 2\pi + 2k\pi \rangle;$$



2)

$$|\cos x| = \cos x \ \text{ za } \ x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right],$$
$$|\cos x| = -\cos x \ \text{ za } \ x \in \left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle,$$
$$\operatorname{tg}|x| = \operatorname{tg} x \ \text{ za } \ x \in [0, \infty\rangle,$$
$$\operatorname{tg}|x| = -\operatorname{tg} x \ \text{ za } \ x \in \langle -\infty, 0\rangle;$$

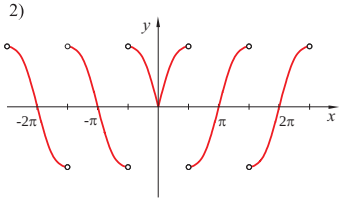
$\underline{x \in [0, \infty\rangle}$:

$$f(x) = \sin x \ \text{ za } x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right],$$
$$f(x) = -\sin x \ \text{ za } x \in \left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle;$$

$\underline{x \in \langle -\infty, 0\rangle}$:

$$f(x) = -\sin x \ \text{ za } x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right],$$
$$f(x) = \sin x \ \text{ za } x \in \left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle;$$

2)



3) Uvjet: $\sin x \neq 0$, $x \neq k\pi$, $k \in \mathbf{Z}$;

$$\sin|2x| = \sin 2x \ \text{ za } \ x \in \langle 0, \infty\rangle,$$
$$\sin|2x| = -\sin 2x \ \text{ za } \ x \in \langle -\infty, 0\rangle,$$
$$|\sin x| = \sin x \ \text{ za } \ x \in \langle 2k\pi, \pi + 2k\pi\rangle,$$
$$|\sin x| = -\sin x \ \text{ za } \ x \in \langle \pi + 2k\pi, 2\pi + 2k\pi\rangle;$$

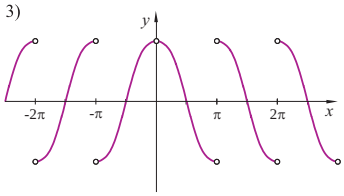
$\underline{x \in \langle 0, \infty\rangle}$:

$$f(x) = \frac{\sin 2x}{\sin x} = \frac{2\sin x\cos x}{\sin x} = 2\cos x \ \text{ za } x \in \langle 2k\pi, \pi + 2k\pi\rangle, \ x > 0$$
$$f(x) = \frac{\sin 2x}{-\sin x} = \frac{2\sin x\cos x}{-\sin x} = -2\cos x \ \text{ za } x \in \langle \pi + 2k\pi, 2\pi + 2k\pi\rangle, \ x > 0$$

$\underline{x \in \langle -\infty, 0\rangle}$:

$$f(x) = \frac{-\sin 2x}{\sin x} = -\frac{2\sin x\cos x}{\sin x} = -2\cos x \ \text{ za } x \in \langle 2k\pi, \pi + 2k\pi\rangle, \ x < 0$$
$$f(x) = \frac{-\sin 2x}{-\sin x} = \frac{2\sin x\cos x}{\sin x} = 2\cos x \ \text{ za } x \in \langle \pi + 2k\pi, 2\pi + 2k\pi\rangle, \ x < 0$$

3)

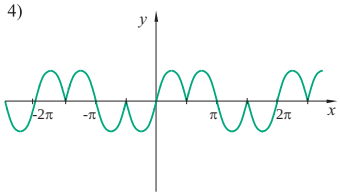


4) Uvjet: $\cos 2x \neq -1 \implies 2x \neq \pi + 2k\pi,\ \ x \neq \dfrac{\pi}{2} + k\pi$;

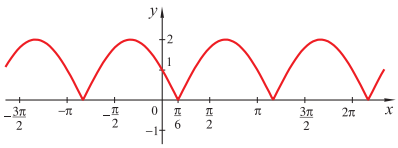
$$f(x) = \frac{\sin x + \sin 3x}{\sqrt{2\cos 2x + 2}} = \frac{2\sin 2x \cos x}{\sqrt{2(\cos^2 x - \sin^2 x) + 2}}$$
$$= \frac{2\sin 2x \cos x}{\sqrt{2\cos^2 x - 2\sin^2 x + 2\sin^2 x + 2\cos^2 x}} = \frac{2\sin 2x \cos x}{\sqrt{4\cos^2 x}}$$
$$= \frac{2\sin 2x \cos x}{2|\cos x|} = \frac{\sin 2x \cos x}{|\cos x|};$$

$$|\cos x| = \cos x \ \text{ za } \ x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right],$$
$$|\cos x| = -\cos x \ \text{ za } \ x \in \left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle,$$
$$f(x) = \sin 2x \quad \text{za } \ x \in \left\langle -\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi \right\rangle$$
$$f(x) = -\sin 2x \quad \text{za } \ x \in \left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle;$$

4)



5) $f(x) = |\cos x - \sqrt{3}\sin x| = 2|\dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x| = 2|\sin\dfrac{\pi}{6}\cos x - \cos\dfrac{\pi}{6}\sin x| = 2|\sin\left(\dfrac{\pi}{6} - x\right)|$;



6) $f(x) = |\sin 2x + \sqrt{3}\cos 2x| = 2|\dfrac{1}{2}\sin 2x + \dfrac{\sqrt{3}}{2}\cos 2x| = 2|\dfrac{1}{2}\sin 2x + \dfrac{\sqrt{3}}{2}\cos 2x| = 2|\sin\dfrac{\pi}{6}\sin 2x + \cos\dfrac{\pi}{6}\cos 2x| = 2|\cos\left(\dfrac{\pi}{6} - 2x\right)| = 2|\cos\left(2x - \dfrac{\pi}{6}\right)|$;