

Zadatak 25. Za dane funkcije $f(x) = \log_{\frac{1}{2}} x$ i $g(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$ riješi nejednadžbu $(g \circ f)(x) < 0.6$.

Rješenje. $f(x) = \log_{\frac{1}{2}} x$, $g(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$, $x > 0$

$$\begin{aligned} (g \circ f)(x) &= \frac{2^{\log_{\frac{1}{2}} x} - 2^{-\log_{\frac{1}{2}} x}}{2^{\log_{\frac{1}{2}} x} + 2^{-\log_{\frac{1}{2}} x}} = \frac{2^{-\log_2 x} - 2^{\log_2 x}}{2^{-\log_2 x} + 2^{\log_2 x}} = \frac{\frac{1}{x} - x}{\frac{1}{x} + x} \\ &= \frac{1 - x^2}{1 + x^2} = \frac{1 - x^2}{1 + x^2}, \quad 0.6 = \frac{3}{5} \end{aligned}$$

$$\begin{aligned} \frac{1 - x^2}{1 + x^2} < \frac{3}{5} &\implies 5 - 5x^2 < 3 + 3x^2 \implies 2 < 8x^2 \implies x^2 > \frac{1}{4} \implies |x| > \frac{1}{2} \\ &\implies x > \frac{1}{2}. \end{aligned}$$