

Zadatak 28. Riješi nejednadžbu $(f \circ g)(x) \leq (g \circ f)(x)$, gdje je $f(x) = \log_{0.5} \frac{x}{4}$, $g(x) = 2^{x-1}$.

Rješenje. $f(x) = \log_{0.5} \frac{x}{4}$, $g(x) = 2^{x-1}$

$$(f \circ g)(x) = -\log_2 \frac{2^{x-1}}{4} = -\log_2 2^{x-3} = 3 - x$$

$$(g \circ f)(x) = 2^{-\log_2 \frac{x}{4} - 1} = 2^{-(\log_2 \frac{x}{4} + \log_2 2)} = 2^{-\log_2 \frac{x}{2}} = 2^{\log_2 \frac{2}{x}} = \frac{2}{x}$$

$$3 - x \leq \frac{2}{x}$$

$$3 - x - \frac{2}{x} \leq 0$$

$$\frac{3x - x^2 - 2}{x} \leq 0$$

$$\frac{x^2 - 3x + 2}{x} \geq 0$$

$$\frac{(x-1)(x-2)}{x} \geq 0$$

$$\implies x \in \langle 0, 1 \rangle \cup [2, +\infty).$$